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The Mathematics Teacher

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Two problems illustrating the use of mathematics in modern industry

CHARLES R. HICKS, *Purdue University, Lafayette, Indiana.*

Refreshing and new applications of mathematics are always welcomed by teachers of secondary mathematics. Not many readers of THE MATHEMATICS TEACHER *will have heard of these interesting and usable illustrations.*

MUCH HAS BEEN WRITTEN about the important role a mathematician plays in a modern industrial organization, but the high school teacher often wants to see some specific uses that are made of the mathematics he teaches. The following examples will show the application of elementary mathematics, although more general problems would involve higher mathematics.

Problem No. 1: A manufacturing plant plans to make two models of a new product. Model I is known to produce \$5 profit per unit and Model II, \$10 profit per unit. The problem is to decide how many of each model to make in order to yield the maximum profit. You may say that the answer is obvious: make only Model II. However, the time required to manufacture each model on two machines through which each model must go is less for Model I than Model II. In fact the required times in minutes are given as:

	Model I	Model II
Machine A	3	4
Machine B	2	6

It is also desirable to run no machine over 3 hours or 180 minutes for the complete order.

Solution: If we let x_1 equal the number of Model I to be made, x_2 equal the number of Model II to be made, the profit (P) can be expressed as

$$P = \$5 \cdot x_1 + \$10 \cdot x_2$$

and we want this P a maximum subject to certain restrictions placed on the machines, namely,

total time Machine A is used

$$= 3x_1 + 4x_2 \leqq 180 \text{ minutes}$$

total time Machine B is used

$$= 2x_1 + 6x_2 \leqq 180 \text{ minutes.}$$

We also require that $x_1 \geqq 0$, $x_2 \geqq 0$.

If we consider the restrictions as equalities only and plot on a graph, we get the curves shown in Figure 1.

Here the shaded area represents a region in which both restriction inequalities would hold. For example, at point R: we would make 35 of Model I and 10 of Model II. Machine A would be involved for $3(35)+4(10)=145$ minutes and Machine B for $2(35)+6(10)=130$ minutes, both values being within the allotted time of 180 minutes. However, at point Q: Machine A time $=3(10)+4(30)=150$ minutes (O.K.) but Machine B time $=2(10)+6(30)=200$ minutes, which is over the time allotted to that machine. Hence any combination of x_1 and x_2 lying

in the shaded area will satisfy the given restrictions. Now, how do we choose the combination which will give a maximum profit?

Next we plot profit curves or $P=5x_1+10x_2$ on the same set of axes. Taking various values of P, we get a family of parallel lines with slope $-\frac{1}{2}$. (See broken lines on Fig. 2.) The line which gives maximum profit (P) and is still within the shaded area will determine which combination of x_1 and x_2 will yield the maximum profit. In this case, we see that the maximum profit will be realized where the two restriction lines intersect. We can think of this profit line as moving upward and to the right until it passes out of the shaded region. Thus, at the point M we get the proper combination of x_1 and x_2 for a maximum profit. As this point is the intersection of the two restriction lines, it can be found by solving simultaneously:

$$3x_1+4x_2=180$$
$$2x_1+6x_2=180$$

This can, of course, be done by substitution, elimination or by determinants. Choosing the latter (which should be taught in advanced algebra), we find:

$$x_1=\frac{\begin{vmatrix}180 & 4\\ 180 & 6\end{vmatrix}}{\begin{vmatrix}3 & 4\\ 2 & 6\end{vmatrix}}=\frac{360}{10}=36$$

$$x_2=\frac{\begin{vmatrix}3 & 180\\ 2 & 180\end{vmatrix}}{10}=\frac{180}{10}=18$$

So we conclude that the maximum profit will be realized if we make 36 of Model I and 18 of Model II and this profit will be:

$$P=5(36)+10(18)=\$360.$$

Comment: This problem taken from the field of linear programming illustrates the use of graphs, expressing a problem in algebraic terms, simultaneous equations, determinants, etc. A more involved problem with many models and several machines would require the solution of several inequations, and the techniques of calculus (for maximizing) and matrix algebra are required. However, for a basic understanding of such problems in programming and allocation of goods, one needs the simplified approach of high school mathematics.

Problem No. 2: A design engineer wishes to put tolerances on an over-all assembly of two parts whose given specifications are:

Part A: $2.100''\pm0.004''$

Part B: $1.800''\pm0.003''$

Should the over-all specification be $3.900''\pm0.007''$?

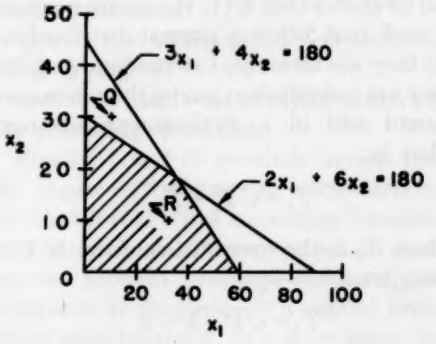

Figure 1

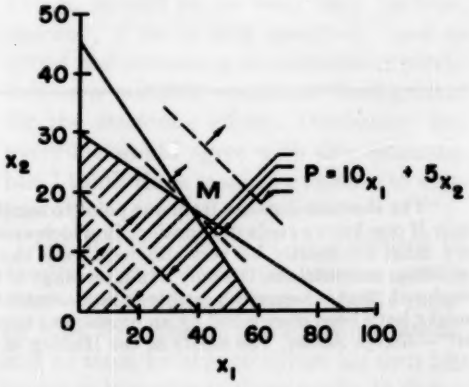

Figure 2

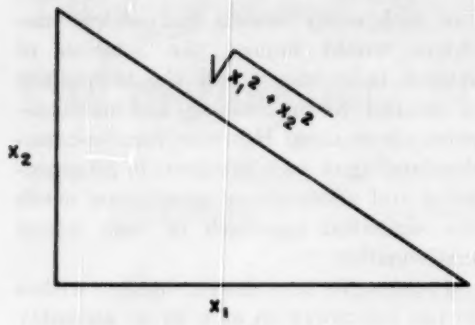

Figure 3

Solution: Certainly the designer is safe in putting a 0.007″ tolerance on the over-all assembly, but from a probability viewpoint, the chances that Part A will measure 2.104″ at the same time that Part B measures 1.803″ is very small. In fact it can be shown that if (1) the measurements of each part follow a normal distribution, (2) they are assembled at random, and (3) they are independent parts, the tolerances should add in a Pythagorean manner. That is:

$$T_t = \sqrt{T_A^2 + T_B^2}$$

where T_t is the over-all tolerance. In this case, then

$$T_t = \sqrt{4^2 + 3^2} = 5$$

and the specifications can be safely set at $3.900'' \pm 0.005''$ with little risk of an assembly exceeding $\pm .005''$.

Comment: The above shows the use of the good old Pythagorean Theorem ($c^2 = a^2 + b^2$) in solving a design problem. Many basic formulas in statistics involve this theorem as we assume independent variables which means the variables are considered at right angles to one another and the sum is the Pythagorean sum. (See Fig. 3.)

An interesting application of this principle made at one of our leading electrical companies was: How to get a 200-ohm resistor with tolerances $\pm\frac{3}{4}\%$. To have one specially made would cost the company about 40 cents. Using the Pythagorean addition of tolerances idea, someone suggested adding two 100-ohm resistors in series, each one being a 1% resistor (i.e., its tolerance band would be ± 1 ohm). The resulting resistance would be

$$200 \text{ ohms} \pm \sqrt{(1 \text{ ohm})^2 + (1 \text{ ohm})^2}$$

$$= 200 \pm 1.414 \text{ ohms}$$

or approximately 200 ohms $\pm 0.7\%$. This is even better than the $\pm 0.75\%$ required and two 1% resistors could be purchased for 1 cent each. The savings to the company using thousands of such resistors is quite obvious.

These examples are intended to show some applications of elementary mathematics in our present-day industrial world.

"The shortest distance from one point to another is a geodetic line, but such a line can be followed only if one knows one's destination, in which case there would be no discovery. The ways of discovery must necessarily be very different from the shortest way, indirect and circuitous, with many windings and retreats. It's only at a later stage of knowledge, when a new domain has been sufficiently explored, that it becomes possible to reconstruct the whole theory on a logical basis, and show how it might have been discovered by an omniscient being, that is, if there had been no need for discovering it!"—*George Sarton*, The Study of the History of Mathematics, *Harvard University Press, 1936, page 19.*

Emotion and thought

ROBERT B. DAVIS, *University of New Hampshire, Durham, New Hampshire.*

That emotions can both facilitate learning and block learning is an accepted psychological fact. The author considers those factors which he feels develop an optimal learning situation in the classroom.

IF I MAY SPEAK of the human mind, somewhat roughly, as a computing machine, then it is certainly a machine which is formidably intricate and incredibly delicate. Moreover, and this is the point which I wish to discuss, it is devastatingly subject to the influence of the emotions. The teacher who is trying to improve the problem-solving ability of a student is consequently involved in the manipulation of an awesome piece of machinery. The result of this manipulation may range all the way from permanent improvement to permanent damage. The matter is worth discussing largely because it lies within our ability to secure a maximum of improvement and a minimum of damage, provided we know how to go about it. But it is not always done the way one might expect. Goals like accuracy and precision must be approached with some caution; under certain circumstances they can become, surprisingly, not the desirable things we usually think them, but rather obstacles and sources of difficulty. This occurs chiefly when these goals are overstressed, or when they are used as the vehicle for expressing such emotions as resentment or hostility.

1. *The subtlety of emotional influences.* Recently I was working to generalize a theorem I had just proved. I found myself not really trying. I was not really trying because I suspected that no improvement was possible. Probably, also, I was not really trying because I wanted to be done with the matter, give the existing version to the typist, and get on with something else. Finding an improvement would force me to alter everything that had been written.

I did, finally, establish the extended version of the theorem. But this became possible only after I had settled three preliminary matters:

First, I had to recognize that I was *not* really trying.

Second, I had to make myself believe in the *possibility* of success.

Finally, I had to persuade myself that the improved version was worth striving for, that it really *was* something I *wanted*.

This influence of the emotions on one's rational thought processes—especially as it appears in students—is a central problem of good teaching. As a sheer guess, we might estimate that the teacher is well advised to devote one-tenth of his effort to the objective presentation of material (which should be an easy task for him, anyway, if he is well qualified), and to spend the remaining nine-tenths in establishing a suitable emotional background for the student's efforts. Obviously, not everyone would agree with this estimate, but I think there is much evidence to support it.

The teacher who is to be cognizant of emotional influences must possess, among other qualifications, these two: He must be able to perceive the student's needs, and he must be able to adjust his own behavior in response to these needs. In doing this, the teacher's chief obstacle is the

large heritage of attitudes which are inappropriate to the situation. On the other hand, his chief asset is his understanding of his own mental processes and feelings and his knowledge of what works well in his own case.

The important influences in the teacher-student relation are very *small* things: an inflection in the voice at the end of a question, or perhaps even a glance, with no words spoken. We must be careful not to become so concerned with the gross, macroscopic phenomena that we overlook the nuances. Which teachers terrified us, when we ourselves were students? Did it depend upon what they actually said, or upon the *way* that they said it? Or even upon the way that they acted and looked, when they weren't speaking at all?

As a further example of the sort of thing we are dealing with, it is worth noticing how many different matters are involved in "wanting to learn": wanting to win the teacher's approval, wanting to avoid punishment, wanting to stand on one's own feet without the teacher's help, wanting to be able to defend one's own ideas against any criticisms of the teacher's, wanting to develop self-confidence, wanting to contribute a new idea, and many other things. I might add: wanting to satisfy one's own sense of what is right. The diversity and depth of these phenomena is never exhausted, however long and carefully the teacher may observe the student's motives. He who is not alive to the subtleties of student desires and student motivation—indeed, he who is not thrilled and intrigued by them—has little likelihood of being a good teacher. But the phenomena are of the greatest subtlety, and many teachers find them easy to overlook. He who thinks of motivation as rooted in good grades and later earning power is a poor observer indeed.

2. *Student needs and teacher attitudes.* Perhaps we should consider a few typical needs of students, and a few of the attitudes that frequently appear as obstacles to the teacher. This is no easy matter. It is this very subtlety of the phenomena which confounds us. The inter-relations between two people consist of a perfectly evident verbal part, and of a nonverbal part that is not at all evident. Unfortunately, the "invisible" nonverbal part is usually the more decisive. However much we dislike someone, it is not at all easy to say exactly *why* we dislike him. If we are strongly attracted to some other person, it is not to be explained, ordinarily, on the basis of something he has *said*, but rather because of something he has communicated without using the literal meaning of words.

Let us give an example. In the final scene of the recent movie version of Herman Wouk's *Caine Mutiny*, the captain and a young officer named Keith are standing on the bridge of a minesweeper, about to take her out to sea. Keith has been distinctly arrogant, and remarkably incompetent, in his past performances. As if this were not enough, he has made matters considerably worse by voicing a dislike for the captain, and by becoming involved in a mutiny. Keith, in short, is a problem. Now for the scene on the bridge: it is simple in the extreme. The captain looks at Keith. "Keith," he says, and pauses for a memorable piece of nonverbal communication, "take her out!"

By facial expressions, by tone of voice, by whatever other subtle mannerisms—as well as by the act itself of entrusting the ship's safety to Keith—the captain has said, in effect: "Go ahead. You will find yourself. If you try to take the ship out, nothing terrible will happen. I trust you; you can afford to trust yourself a bit."

Undoubtedly, any exceptional nervousness on the part of the captain would have been communicated to Keith and would surely have impaired his performance. Any insincerity, ambivalence, resentment, or similar feeling would also have been communicated. Each of us has everyday experiences which demonstrate how easy it is to "get rattled"—often in response to someone else's anxiety. The teacher's feel-

ings are likewise transmitted to the student, and have the strongest influence on how well he likes a subject, how hard he works in it, the quality of the work, and his use of the subject in years to come. This last point alone would suffice to direct a great deal of attention to the communication of attitudes between teacher and student. After all, in later life, the student can go out of his way to make use of a certain subject, or he can go out of his way to avoid it; if he does use it, he can do so creatively, or he can do so in a thoroughly routine way. Much of this is determined by minute nuances experienced in the classroom.

In one of my own classes I recently had a student who presented a picture rather like Keith. I am sure this student disliked teachers. He had accumulated a long record of failures in mathematics. I believe he found the subject incomprehensible. His own ideas of what was important within a problem had, I suspect, never seemed to coincide with the ideas of his teachers. (Incidentally, I might add that the teacher who is not able to explain his ideas so that the student willingly accepts them, and understands them, but who is nonetheless prepared to insist upon these ideas, is one of the major sources of resentments and feelings of inadequacy among mathematics students.) He appeared to be left with the feeling that it was impossible ever to please the teacher, that mathematics was a magical subject where any reasonable decision would later turn out to be wrong, for some reason of the most unpredictable sort. It doubtless seemed to him to defy understanding. Mathematics looked to him like a combination of witchcraft and the capricious misuse of authority.

Toward the end of the term he began to work diligently, if somewhat uncomprehendingly. I take some credit for this spurt of activity. I imagine I had made him feel that success was a more *real* possibility than he had formerly thought it to be. On one occasion he volunteered to go to the board and solve a problem. This already indicated a great increase in his belief that "nothing terrible would happen." By this I refer especially to the nonverbal relations between him, the class, and me. The achievement which I claim for myself is that I had made him feel, after a term's contact, that my *attitude* toward him would continue to be friendly and encouraging, whatever came of his efforts at the blackboard. This, of course, was a nonverbal matter. It makes little difference what the teacher *says;* the crucial thing is his manner.

When this student went to the board, he was noticeably anxious. He worked most of a difficult problem with fair accuracy (getting almost the right answer), though he omitted some parentheses and made a few similar errors. When he had finished, I told him that I thought he had used a lot of good ideas (which was entirely the truth, and may consequently have sounded sincere). Then I asked him, without specification, if he had written exactly what he had intended in each of the lines of work. With no further comment from me, he turned and corrected most of his minor errors.

This is in many ways the crux of the matter. By building self-confidence, by pointing out the positive paths to achievement, by encouraging faith in the possibility of success, we do the optimal job of teaching. What we really have to do is to show each student that something he *wants* can be achieved *by his own efforts* in a certain direction. The use of negative inducements, threats, intimidation, or ridicule is a more dangerous and far less satisfactory approach. Veiled threats are no better. The trouble with inhibitions, as everyone today seems to know, is that they have an alarming tendency to spread. If we say "don't" to someone often enough, he soon becomes unable even to do those things which *are* permissible. (One encounters students of piano who have been told so frequently, and with such convincing feeling, not to make noise and disturb

the rest of the family, that this has finally become a factor in their striking wrong notes during a performance. After all, music is a communication between performer and hearer, and if the hearer has habitually interpreted it as a hostile communication the player may gradually come to accept this evaluation, and to develop the guilt feelings which one would then expect.) In mathematics, I have seen students reject a simple and satisfactory solution to a problem, because they have acquired the feeling that things just are not that easy. Anything that nice must be immoral!

I can summarize the question of student needs by saying that the student does need someone to point out the paths to satisfaction; he does not need someone to stand by and threaten or intimidate. Every human being is trying, in a profound sense, to do his best. We can be most helpful by keeping open at least some paths to satisfaction, by showing that these paths are a real answer to his needs and desires, and by not confronting the student with desperate and unsolvable dilemmas.

3. *Attitudes which are obstacles.* Many teachers overemphasize the demands of accuracy and precision. They can make a misplaced minus sign appear as the epitome of total failure. They can create a state of anxiety in the student, until he is unable to approach any problem in mathematics without a terrifying sensation of tottering insecurely at the brink of a bottomless pit of absolute incompetence (not to mention ridicule) into which he may at any instant be pushed by the fatal error of misplacing a decimal point. I do not mean to counsel indifference to the need of obtaining a correct answer. The point is that one is *more likely to get* the right answer when one is relaxed and self-confident than when one is anxious and insecure.

In the hope of making these ideas more concrete, I should like to refer to a recent article by Catherine Meehan.[1] This article gives something of a converse to the point of view which I am describing. Her point of view is that, by presenting mathematical techniques, the teacher influences the attitudes of the student. Our present interest is in the converse: the attitudes of the teacher are communicated in a subtle, nonverbal way to the student, and give rise to certain definite attitudes in the student which have a decisive influence on his problem-solving ability. I should like to join Miss Meehan in emphasizing that the flexibility or dogmatic rigidity of the graduate, his creative zeal or his apathy, his responsiveness to new ideas or his rejection of them, all date back in large part to his student days, to the subtle examples set by the attitudes of various teachers.

Miss Meehan writes:

> . . . we meet local employers, day after day, who beg us to teach *attitudes*, good old-fashioned *attitudes*. If this could be done by means of another course, someone would have tried it before now. It used to be done in the regular classroom. It is still being done in mathematics classes led by alert, enthusiastic, well-prepared teachers. . . .[2]

I hope that I may be forgiven my use of a couple of excerpts as a springboard to plunge into my own remarks. In regard to this first quotation we might emphasize these points.

1) Attitudes certainly are taught by countless *little* experiences, all day long, in class and out. This is the very aspect of education which we wish to emphasize. A teacher who counsels self-confidence in all of the big things, but who induces diffidence in all of the little things, will have a harmful effect in the long run. Perhaps, for example, he is inclined to interrupt impolitely at times when the student is talking, implying a real lack of respect for the student's presentation—perhaps also for the student.

2) The potentialities for inducing good attitudes or bad are tremendous. Many adults possess excellent thought processes which they owe, in part, to certain of their

[1] Catherine Meehan, "Mathematics and the Development of Good Citizens," The Mathematics Teacher, Vol. XLVII (April 1954), pp. 226–230.

[2] *Ibid.*, p. 226.

teachers. Alternatively, many adult cases of anti-intellectualism, lack of self-confidence, over-expression (or over-inhibition) of hostility, distaste for certain subjects, or failure to appreciate abstract thought can be traced back to teachers who have been instrumental in establishing such patterns. As an example, we might cite the case of the teacher who always insists upon being right, and who may thereby create in students a "what-does-it-get-you" attitude toward logic and rationality. Logic may appear as an inadequate defense against the capricious authority of the teacher. The good teacher always tries to make it clear whether he is stating his opinion, or is dealing with objective fact. As much as possible, he guarantees the student a secure refuge in logic. Contrariwise, I have heard poor teachers brush aside the student's logic even when it was largely correct. One might say: in a good school, a freshman with a rational line of reasoning will prevail over a dean who cannot thus support his conclusions. In a poor school, the dean is always right. What place has logic in such a school?

3) As we have mentioned earlier, it is not what you do, but the way that you do it. The attitudes of the teacher are decisive, for it is these which influence the attitudes of the student. The question is not, for example, whether the teacher *tries* to state the need for logic, but rather whether the teacher himself trusts his own logic, and genuinely believes in the student's ability to be logical. If the teacher is over-controlling, and interferes too much in the student's thought processes, the student has no opportunity to develop his own logical facility. That the over-controlling teacher makes speeches extolling logical thought means little if he continually interferes, as some teachers do, in all of the student's attempts to think for himself.

Permit me to interrupt myself. It occurs to me that I may not be understood, because the reader may be saying to himself: "But, after all, isn't the internal logic the important thing?" Yes, of course. That is to say, it is the important thing to the mathematical structure itself; it is the important thing once it is all set down on paper. But there is a stage, or even many stages, prior to that. One must select strategies, or lines of attack. One must recall this, and see the possibility of using that. In later life, one must choose between various problems, and decide which are feasible and which are not. If a line of attack is not leading to success, one must choose between abandoning it or persevering, and extreme error in this direction is not uncommon; often people have long continued attempts in a hopeless direction, or have left an approach just short of victory. One must decide when to resort to a search of the literature, and when to figure the matter out for oneself. Often there are many different *types* of solution, and one must decide which type to look for, as in choosing between finding a primitive function in integration, or resorting to numerical methods. For the student, who has not yet achieved such a high level of integration of his knowledge, there is often the choice of whether to use the method learned in trigonometry, and when to use the method learned in analytic geometry. One must sometimes decide either to try a certain change of variable, or else to reject it as unlikely. One can give literally millions of examples; indeed, one cannot solve even the simplest trigonometric identity without making many choices of this sort. I believe this is one of the functions of *inspiration* in teaching—to provide insight into how to make these decisions. This is done chiefly by the example of the teacher, and is one reason why I believe that it is virtually impossible to teach mathematics unless one is really extremely proficient in the subject. I know many teachers who are a few pages ahead of the students in the textbook, and who are known for the clarity of their lectures and are considered adequate teachers. I feel they are not. They are usually incapable of showing the

student, the important example of a man who knows how to make all of these choices, and without this, teaching is meaningless.

The reader may further ask: "Still, isn't student performance a question of aptitude or ability? If the student hasn't got what it takes, what is the teacher to do then?" Well, there is no doubt that there is such a thing as ability, or aptitude, or intelligence, just as there is such a thing as body weight. But just like body weight, this is relatively constant just so long as we do not make any intelligent effort to change it. The student's aptitude in a subject is the historical summation of his experiences relating to that subject. It is not even necessary to argue over the question of heredity. There is so much plasticity and potentiality for change in virtually every student that hereditary differences, even if they do exist, are not ordinarily the decisive factor. Heredity, or aptitude, or intelligence is usually neither more nor less than the excuse which the teacher offers when he has failed to understand the needs of a certain student or has been unable to meet these needs. Most "student" failures can be traced back to some adult, often a teacher though not always the current teacher. Sometimes one must look to the teacher of an earlier course who has left a residue of distaste, lack of self-confidence, indecision, and anxiety.

But after all, how can I say to the teacher: This is your job; look to it! Many good teachers have recognized this as their job, as far back as we know the history of teaching. To teach is to inspire. Many teachers (whom I unhesitatingly label inadequate) say: No, it is not my job. My job is to present material; if the student can get it, well and good. But if he fails, do not blame the failure on me.

And I would be the first to agree that it is not an individual matter. The cumulative effect of previous teachers may be more than one can counteract. It is precisely for this reason that the criteria of teaching are so complex. A teacher may say he has taught successfully this way for years; ah, but what sort of problem has he been leaving for the teachers who had to deal with these students in following years? Did he teach the students to think for themselves? Did he teach them to like the subject, and to want to work hard in it? Did he teach them to approach problems with self-confidence? Or perhaps just the opposite?

What of the person who, in later life, carefully avoids the use of mathematics as much as possible? Can we say that his mathematics teachers were successful, simply on the basis that he was once able to solve quadratic equations? The simplicities of examinations and grades are no substitute for the complex criteria of later life, and teachers, if not students, must be judged by these long-range criteria.

Suppose that we consider a second excerpt from Miss Meehan's article:

> I believe that we give our attention to developing the individual as a person who will be able to really *live*—happily, successfully, and peacefully in our democratic society—and who *will know* and will fight fiercely when his standards are imperiled. . . .
>
> What are *some* of these standards?
>
> 1. *Accuracy*—By this standard we hope to develop attitudes of promptness, precision, honesty, etc.
>
> When Sammy, in the early grades, is given this problem, "There were five birds on a branch of a tree; two flew away; how many are still on the branch?" his answer must be "three" and only "three." "Two" is not acceptable on the theory that if there are three left there are certainly two! What kind of a city treasurer or a club treasurer or a banker would Sammy make if we took "two" for his answer? Equivocation is not honest and we just won't have it.[3]

One could discuss this excerpt at great length; let me settle for two remarks:

First, suppose that Sammy does in fact argue that there are two birds left. I think we would make a mistake if we brushed aside his logic here. We would run grave risk of forcing him to profess a belief in *our* logic, while he acquires more than a shade of doubt as to his own ability

[3] *Ibid.*, pp. 226–227.

to look at the phenomena and follow out their intrinsic logic himself. Is he wrong in saying that there are two birds left? As a matter of fact, he evidently is not; he has seen a real truth concerning number which we should not brush aside. Perhaps he has been more astute and more profound than we have been. He has pointed up a weakness, not in his logic, but rather in our question. "Two birds are left" is a true statement! What we are really asking for is the strongest statement of this kind that can be made, and "Three birds are left" is also a true statement, is stronger, and is in fact the strongest true statement, hence the answer. We would do Sammy an injustice—hardly the example which we mean to set—if we made him feel that our question was properly posed and his logic was fallacious. In truth, our question was vague, and he responded with a correct statement whose logical validity is unassailable. There *are* two birds left. That there are three implies *a fortiori* that there are two.

Sammy has looked at the situation clearly and realistically. Do we wish to brush this aside? Do we *want* him to take our word for it? If he does, he runs grave risks for the future. What criteria will be left him later on? Is he to try to remember what people have *told* him is true, or is he to try to figure it out for himself?

There is a digression here which I cannot resist. Miss Meehan argues that we want Sammy to be a good club treasurer. Well, if we are to take ultimate social values into account, as Miss Meehan essentially implies, is a numerical error in arithmetic so much worse than pathological gullibility, which we seem to instill in many a student today? Neither is desirable but in social terms, the mathematics teacher might well give a thought to the question of whether he is preparing his students to believe the worst lies of propagandists, because he has taught them to disbelieve the evidence of their own senses and their own logic, and to accept a logic which is foisted upon them from without.

But that is really irrelevant to my central theme. I claim that we must have a care lest Sammy soon find himself unable to do mathematics, irrespective of whether or not he becomes grist for the mills of the propagandist. There is no foundation for science other than logic mastered within one's own mind. There is no rote approach to science. There is no acceptance of external logic. The acceptance of external logic is itself illogical.

My second comment is this: "Promptness, precision, honesty, etc." are worthwhile virtues, but we must use the words with caution, lest we be utterly misunderstood. There is a kind of compulsive promptness which is virtually nothing more than a weapon with which some individuals belabor an easy-going world. Is our ideal the rigid individual who reproaches acquaintances when they are two minutes late for a luncheon engagement? Or the individual who demands absolute punctuality from his employees? I should say these are hardly the ideal; the ideal person is flexible, adaptable, easy to get along with, and reasonably forgiving and generous. Training in "promptness, precision, honesty, etc." is a strong medicine which we must take care not to administer in over-large doses. To do so is detrimental to the way the student relates to other people, is harmful to his own creative thought processes, and exerts a pernicious influence on the way he feels about himself.

I have mentioned various things which I feel the good teacher does or does not do. It is worth adding one more: I believe every really good teacher is frequently struck with a sense of awe at the consistent logic of nearly all of his students. (True, this may be most remarkably obscured by the fact that the student did not trouble himself to glance at pages nine and ten of the text, and so on.) Students, youngsters, even infants show an almost incredible natural gift for logic. I recently watched a nine-month-old baby trying to escape past a gate that had been put in place

with him in mind. In back of him was an unfastened gate of identical construction, not needed at the moment. After unsuccessful efforts at moving the fastened gate, the baby returned to the unfastened gate, which he could move with ease. He repeatedly went from one gate to the other, clearly attempting to use the one as an example of how to solve the problem of the second. I am awe-struck by the cleverness of such an approach.

Many remarks along the present lines could be appended to various other excerpts from "Mathematics and the Development of Good Citizens." I may as well confess that, taking that article as a whole, I feel a lack of sympathy with the tone of it all, though perhaps I misunderstand the true meaning. To arrive at a proper balance in educational matters requires constant discussion, a constant meeting of different minds. Teachers must be especially alert to recognize any harmful tendency in themselves toward being over-controlling or overly rigid. The teacher is so frequently right, and stands so unmistakably in a controlling position, that he must make great efforts to yield to the correct logic of students, and to permit them the generous amount of freedom which is necessary for learning. Students must be allowed to make mistakes, to try out their own ideas, to think for themselves. Good teaching cannot be built on the attitude that the students are stupid. In point of fact, the logic of students, as has been remarked above, is nearly always at least partly correct. The good teacher builds on the correct part, whereas the poor teacher attacks the weaker part. Hence the good teacher leads the student along the path to self-reliance and self-confidence, while the poor teacher induces feelings of inadequacy and distaste for the subject.

Unfortunately, a correct evaluation of oneself is extremely difficult. Many of the best teachers struggle prodigiously for self-improvement; many poor teachers, over-controlling and rigid, continue complacently in the same course year after year. It has been wisely said that one must distinguish twenty years of experience, which some teachers have, from one year of experience repeated twenty times, which other teachers have. Few poor teachers show any real awareness of their inadequacy; rather, they have countless rationalizations to justify themselves.

4. *Two remarks.* The reader who is familiar with certain sociological critiques of the culture of the United States (and perhaps especially of New England) may have recognized that many of my comments have been expressed in other language by saying that we have inherited a culture which, if it has a major defect, suffers chiefly from this: it *prevents* too many things, in the general sense. It is itself over-controlling, overly rigid. It seeks to mold youth too much, and to let it grow too little. Somehow this all adds up to an effect of stifling creative expression. Mathematics teaching might be said to tend to reflect this cultural characteristic. It would not be appropriate to consider the issue on broad sociological grounds in this article, and I do not mean to do so. Right within the needs of the creative teaching situation itself we can find ample motivation for encouraging individuality and *not* emphasizing routine acceptance of externally imposed logic.

It occurs to me that I have not used the word *rote* in this article—not yet, in any event. By *rote* teaching I mean the presentation of facts and techniques which are simply to be memorized by the student, with perhaps a certain amount of practice on some of the techniques. This, I claim, is not teaching at all. At least, it is not the teaching of mathematics. Mathematics is not a tool. It is a way of thinking which is creative, flexible, perceptive of reality, and based upon certain good judgment as to what may be eliminated, and what must be retained, in the process of abstraction. It recognizes objectives, and selects strategies. *None of this is taught by rote presentation.* That some teachers will

tell me they can achieve high quiz grades by rote teaching is irrelevant. They have taught all of the inessentials, they have tested all of the inessentials, and they have found their students well versed in all of the inessentials. What has that to do with what I have to say?

5. *Summary.* But to return to my basic theme. Talk about teaching has a tendency to be unsatisfactorily vague, and I would wish to be as concrete and precise as the nonverbal nature of the subject will permit. My chief remark is that rational thought processes are under the influence of emotional factors. This is not a slight influence. On the contrary, emotional factors have a decisiveness which the teacher cannot afford to ignore. If one looks at a school, the over-all pattern of students doing well in courses they like, badly in courses they dislike, and so on, is the first thing that hits the eye. The cumulative effect of this sort of preference, over many years, gives rise to what we call aptitude for one subject, or lack of aptitude for another. (I do not, of course, mean to restrict attention to experiences within the classroom only, nor even to the experiences of school years alone. The bases of aptitudes, as the toy-makers have lost no time in pointing out, extend far back and into experiences of all sorts. The teacher is only one influence, but he is *one*.)

I wish the classroom experience were not so intangible. It is a most refractory matter for discussion. I should like to give an example of some instance where a teacher has, in a subtle and nonverbal way, exerted an influence toward creative thought in students, or, perhaps, an instance of the opposite sort of thing. As a matter of fact, fate has just handed me one example, and, inadequate as it is, let it do. I just interrupted this article to meet a class for the first session of the term. One problem was checking students' names against class cards. Now, the pronunciation of students' names is one of the less certain and predictable phenomena of the universe, and the teacher is not unlikely to be occasionally in error. Just how he handles this situation, the extent to which he encourages students to feel free and secure in correcting him—after all, here is one case where the student is the absolute authority—the way he responds when students do criticize him (and they may not do it gently) sets a good deal of the tone of the class, for that day at least. Or perhaps I should say, this is a very little thing. It is one case where the contest between power and truth is being fought, in the subtlest of ways. The basic principle of psychology, however, seems to be that everything is the outcome of the summation of all of these *little things*. They are small tokens, but they add up to a potent currency.

In case my meaning is not clear, I might add that the mature teacher, who wishes the student to learn to stand on his own feet, will show a generous ability to accept criticism, and will not respond by threatening the student in any way. Do we want the student to think for himself, or don't we? If he is right and we, the teacher, are wrong, we cannot ever be too ready to admit it. The more willingly we do it, the better.

What can be done in the classroom, to improve student performance, is in its way as definite and as important as any of the phenomena of science and art. Unfortunately, it is sadly neglected. I cannot say there is anything like the optimal amount of freedom, democracy, consistent objectivity, and inspiration in the average classroom. Too often one sees a teacher who insists upon being right, who is unreceptive to new ideas, who is unperceptive of the subtler needs of students, or who lacks the vital interest in his subject matter which is necessary if he is to inspire his students. The teacher is rarely able to evaluate himself accurately, especially in view of the nonverbal nature of the subject; though his words be generous and accepting, it mat-

ters not—the real question is whether or not his manner is threatening.

The constructive use of atmosphere to enhance problem-solving ability is strikingly akin to the psychoanalyst's use of "conversation" in treating neuroses. In either case, one is amazed at how much can be done, and by such subtle influences. It is to be regretted that the present-day acceptance of the psychiatrist's "conversational" methods as something definite and real (and even dramatic) does not extend more fully to include the similar techniques and problems of the teacher.

One can hope for the day when a profound understanding of emotional dynamics will be recognized as an ordinary tool of the teacher's trade, virtually on the same level as the ability to speak his or her native tongue. The teacher *must* be alert to the needs and attitudes of the student, and must be able to modify these attitudes, or respond to these needs, wherever the occasion arises. If the teacher is less than this, he is nothing more than a textbook, and may strongly influence the student away from creativity and toward sterile routine.

A foundation-supported conference for mathematics and science teachers

Secondary school mathematics teachers are encouraged to apply for the $200 fellowships which are being made available for 32 teachers who will participate in an interesting and unusual "Research Team" conference at San Jose State College this coming summer.

At the conference, mathematics teachers will join with science teachers in a two-week session, August 12–26, devoted to the mathematics phases of the secondary school sciences.

The "mathematics" nature of this conference is an outgrowth of the "Research Team" conference held at Oregon State College under the auspices of the Future Scientists of America Foundation and the Crown-Zellerbach Foundation during the summer of 1954. At those meetings, science teachers explored research activities and noted the vital part that mathematics played in scientific procedure. Accordingly, the sponsoring agencies looked ahead to a mathematics-science program in which the mathematics teachers could assist in the interpretation of mathematical implications and, at the same time, become more aware of the scope and nature of mathematics needed in science classes.

Thus, The National Council of Teachers of Mathematics is cooperating as the Future Scientists of America Foundation joins with San Jose State College and the Crown-Zellerbach Foundation in conducting this program.

The National Council of Teachers of Mathematics has endorsed the conference through its Board of Directors, and The National Council Committee on Cooperation with the Future Scientists of America Foundation has recommended participation by The National Council and by individual secondary school mathematics teachers. One of the important new features of the conference will be a co-director from the field of mathematics.

Advance publicity for the conference states:

"Data will be gathered by visiting university, government, and industrial research laboratories and interviewing chemists, geologists, physicists, mathematicians, biologists, and other scientists. These visits will allow the participating teachers to see the mathematical skills and abilities of successful scientists. Work and study groups will then extract from these interviews the implications that are appropriate for the high school scene. In turn, these implications will be translated into new high school science-mathematics problems and exercises.

"Selection of participants will be based upon the applicant's ability to share in the development of stimulating and practical science and mathematics exercises for junior and senior high school boys and girls. To insure adequate geographical distribution, eight teachers will be selected from each of the states of California, Oregon, and Washington; and eight from the states of Arizona, Idaho, Nevada, and Utah combined.

"Applications will be accepted any time before May 1 from science and mathematics teachers in grades 7 through 12 in the above states."

Application forms may be obtained from the Future Scientists of America Foundation, National Science Teachers Association, 1201 Sixteenth Street N.W., Washington 6, D. C. Applicants should have their principals and superintendents forward letters of recommendation, touching upon qualifications of the applicants.

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Uncle Ed juggles with figures

G. A. GETTY, *Penney Farms, Florida.*

Some recreational mathematics which will interest many junior high and senior high school students.

WHEN MY BROTHER Bob and I were in high school, Uncle Ed, my mother's younger brother, was a frequent visitor at our house. Bob and I were always glad to see him. He was always full of fun and would talk to us boys as much as he did with the older folks. Moreover he delighted in puzzles of all kinds, and was constantly springing one on us. It was Uncle Ed who awakened our interest in crossword puzzles, an interest that stays with us to this day. One day, when I had been grouching about mathematics and the testy martinet who was teacher of that subject in our school, Uncle Ed exclaimed, "Mathematics? Why Art, that is just about the finest and most useful subject in the whole curriculum, and also the most fascinating."

He paused for a moment and looked at the ceiling with a far-away expression as though he were trying to recall something that he had almost forgotten, and then said, "Each of you boys get a pencil and a sheet of paper. I'm going to show you a little stunt that I call juggling with figures. Perhaps you have seen a professional juggler tossing three or more balls into the air at the same time, and keeping them going for several minutes. Well, I am going to toss you a group of figures, and try to do a little juggling with them."

When Bob and I had seated ourselves at the library table, each provided with pencil and paper, Uncle Ed seated himself in the big easy chair, leaned back, closed his eyes, and said, "I'm going to dictate to you nine lines of figures—eight digits in each line—seventy-two figures in all. Write them down as I dictate them. Keep your horizontal lines straight, and keep the vertical columns lined up also. Write for Line 1 . . . " and he went on, slowly, but without hesitation, to dictate the following table of figures.

1 2 3 6 9 4 7 0
2 4 6 5 8 2 4 1
3 6 9 4 7 0 1 2
4 1 2 4 6 5 8 2
5 3 5 3 5 3 5 3
6 5 8 2 4 1 2 4
7 0 1 2 3 6 9 4
8 2 4 1 2 4 6 5
9 4 7 0 1 2 3 6

When he had finished dictating, Uncle Ed continued speaking, "There you have what appears to be a heterogeneous conglomeration of meaningless figures, but as seen through my closed eyes, it isn't that at all, and I think that I can readily repeat any or all of the nine lines exactly as I have dictated them to you. I might begin with any of them, but suppose we start with Line 4." He then repeated that line just as we had written it down, and continued with lines 3, 2, 1, 5, 6, 9, 8, 7, of course reading them all correctly.

Before Bob and I had a chance to express our surprise at his perfect performance, Uncle Ed went on, "Believe it or not, boys, it would be just as easy for me to read those lines *backward*, as to read them the way you wrote them down. Perhaps the best way to demonstrate that would be to just read the entire table of figures in reverse. I will read each line backward, from right to left, beginning with the bottom line and going up, line by line, to the top, without any break."

And Uncle Ed did just that, beginning with the final 6 at the end of Line 9, and ending with the initial 1 at the beginning of Line 1, and without a single mistake.

Of course Bob and I were astonished. Uncle Ed, however, hardly paused to take breath. He said, "Now we will take a look at the vertical columns. We will start at the left and call that Column Number 1. I will read the eight columns in regular order and from top to bottom." And he did just that.

By this time Bob and I were speechless, but Uncle Ed went right on, "Now Art, mention one of the columns by number, and I will read it from the bottom upward." I said "Column 3," and Uncle Ed immediately called off "7, 4, 1, 8, 5, 2, 9, 6, 3." When Bob's turn came, he named Column 6. Uncle Ed responded, "2, 4, 6, 1, 3, 5, 0, 2, 4." Then Uncle Ed said, "Art, add up column 3, and I think you will find the sum of the nine digits is 45. And Bob you add up column 6, and your answer will be 27." We added our columns and found the answers just as Uncle Ed had said. Then Uncle Ed said, "One of you name one of the horizontal lines," and I said "seven." "Add the eight digits in that line, and you will get 32 for your answer." We soon verified his statement.

"That about ends my little stunt of juggling with that lot of figures," said Uncle Ed, "but there are a number of remarkable features about this apparently heterogeneous conglomeration of meaningless figures that I want to point out to you. You will notice that the last four columns are the same as the first four columns but *inverted*. Column 1, read downward, consists of the digits from 1 to 9 in regular order. Column 5, read from the bottom upward, consists of the same digits and in the same order. If you scan column 3 closely you will find in it the same nine digits, but arranged in different order. Column 3 is a perfect progression by 3's,—3, 6, 9, 2, 5, 8, 1, 4, 7. You need only to remember your multiplication table for 3 and run down its right-hand column. Column 7 read from the bottom upward is the same beautiful progression. Inasmuch as each of these four columns is composed of the same digits, each of them adds up to the same sum, 45.

"The other four columns are composed of an entirely different group of figures. Column 8 starts with 0, repeats 2 and 4, but has no 7, 8, or 9, in it. The digits occur in their regular order, or if you prefer to look at them that way, in three series of three digits each—0, 1, 2; 2, 3, 4; 4, 5, 6. Column 4 is identically the same as column 8, but in reverse. Column 2 contains the same digits, but here the arrangement is different, falling into a pattern of three progressions by two—first three even numbers, 2, 4, 6—then three odd numbers, 1, 3, 5—and last, three even numbers, 0, 2, 4. Column 6 is the same as column 2, but inverted. I have never tried to memorize these *figures*, but have clearly fixed these *patterns* in my mind, and the figures fall into place automatically. Inasmuch as columns 2, 4, 6, and 8 are composed of the same digits, each adds up to the same sum, namely, 27. I may as well say now that I asked you to add but one of the horizontal lines, because each of the nine lines produces the same total—32.

"I want you boys to note that all that I have pointed out to you about the vertical columns is merely incidental. I did not dictate the columns to you. I dictated the horizontal lines. In fact, one day when I was trying to find some simple method of dictating a table of figures that I would be able to recall, and finally evolved a simple formula that would do the trick, I gave no thought to the columns, and did not discover their beauty and symmetry until some time later. Now the question that I am putting to you boys is this: How did I construct those nine lines so that I could recall them and read them forward or backward, today, tomorrow, or a year hence? There they are, on the papers before you. If you can discover the key to the mystery before I come again, I will give each of you fifty cents; if you can

give me the figures as I have dictated them to you, and repeat them backward, you will each get a dollar. I have dictated them many times to various groups, and never has anyone discovered my *modus operandi*. I hope you boys will find the answer to the riddle." He then said goodby, and left.

About a week later Bob and I returned from school one afternoon and found Uncle Ed chatting with mother in the library. "Hello, Jugglers," he greeted us. "How much do I owe you?" We told him that we had spent considerable time and effort to solve the puzzle he had given us, but without success. "Well that's just too bad," he replied. "We'll have to do something about that. Get the papers with figures you wrote down last week, and also two fresh sheets of paper, and we'll tackle the problem right now."

When we were ready Uncle Ed remarked, "We will not attack the citadel from the front, but approach it from the rear, by the path which I first found to enter this stronghold. Write down at the top of your fresh sheet this simple mathematical formula, *Always multiply the last figure named by seven to get the next two digits.*" When we had done so, he added, "Now go ahead and write Line one."

It took us a few minutes to grasp the idea that we were to multiply the "one" that Uncle Ed had emphasized slightly, by seven, and a few moments more to see that the formula called for a zero before the resulting 7, but after that we had little trouble in multiplying the 7 by 7 to get 49, the 9 by 7 to get 63, the 3 by 7 to get 21, and in a few minutes we had written down the following table of figures:

Line 1—	0	7	4	9	6	3	2	1
2—	1	4	2	8	5	6	4	2
3—	2	1	0	7	4	9	6	3
4—	2	8	5	6	4	2	1	4
5—	3	5	3	5	3	5	3	5
6—	4	2	1	4	2	8	5	6
7—	4	9	6	3	2	1	0	7
8—	5	6	4	2	1	4	2	8
9—	6	3	2	1	0	7	4	9

"That table of figures is what I constructed first," said Uncle Ed, "and I had a bit of fun with it from time to time. You will notice that you cannot dictate more than eight digits in any one line, because in doing so you would be merely repeating figures already used in that line. For *the eighth digit is always the same as the line number.* By the same token, that fact affords a sure check that you have made no mistake in dictating that line. If the eighth digit is anything else than the number of the line, you know at once that your mental arithmetic is at fault.

"I used these lines in various ways, sometimes the first three or four. Sometimes I would choose a date such as 1492, and use lines 1, 4, 9, 2. Other variations will occur to you, as you play with your table of figures.

"Then one day I began to wonder if it would be possible for me to read these lines *backward.* I knew the final figure of each line, could I reverse the process by which the line had been constructed, and read it in reverse? How easy it was to do so, surprised me.

"Take Line 1 for example. The eighth digit is 1. That 1 is the second digit of a multiple of 7, which can be nothing else than 21—three times seven. There are the last three digits—321. The 3 must be part of 63, which is 9 times 7, which gives me two more digits, making 96321. The 9 is part of 49, 7 times 7, making 7496321. And the 7 is the number itself, just 1 times 7 and calls for a 0 in front of it.

"So I tried reading the lines backward, and after a few efforts found I could do it as fluently as I could read them from left to right. It was not until some time later that I began to wonder what the magic 7 had done to the vertical columns, and I was more amazed than you have been at the beauty and symmetry shown there.

"Very soon I decided that if I were going to read the lines backward at all, it would be well to dictate them that way first. The first two columns, as you have written them down today, are merely the

multiplication table for 7, from 1 to 9. Anyone with a mathematical mind would be apt to recognize that fact, and then note the peculiar character of line *five*, with its succession of 35's and thus presently solve the mystery. But with the multiples of 7 reversed the secret is rather well hidden, and even a mathematical mind would require some time to detect it."

For the next ten days Bob and I spent some of our spare time working over the two tables of figures that Uncle Ed had given us, and then one day recited the lines forward and backward to him, though not yet certain about the vertical columns. Uncle Ed was delighted and gave us each a dollar. But that was not the best part of it. "Juggling with figures" had created a keener interest in mental arithmetic, and increased our respect for the multiplication tables and for the amazing qualities of the "Magic 7." It had also demonstrated that it was possible to "have fun with figures."

A foundation-supported conference

Continued from page 142

Other supplementary information and additional references are highly desirable. Science-related hobbies and special interests, experience with sponsorship of student projects, and contacts you may have had with governmental or industrial laboratories should be mentioned.—*Donald W. Lentz, Chairman, NCTM Committee on Cooperation with the Future Scientists of America Foundation.*

Have you read?

ROSSKOPF, MYRON F. "Trends in Content of High School Mathematics in the United States," *Teachers College Record*, vol. 56, December 1954, pp. 135–138.

In the course of a busy day of teaching we seldom take time to look where we have been or where we are going. In this paper, presented before the International Congress of Mathematicians in Amsterdam last September, Mr. Rosskopf has done just that. Under the mental discipline of philosophy, mathematics was in a very secure position. However, the impact of Dewey's and Thorndike's work and industrialization in the United States greatly affected this position of mathematics. The result has been more activity in the mathematics classroom, and more concern for reasonable applications of mathematics. What does such a program do to the content of mathematics? First of all, it builds a general dissatisfaction in the present mathematics program with its compartmentalization and show-drill instruction.

The two courses of action possible seem to be a reorganization which would integrate the present content of high school mathematics or a complete reorganization which would bring into high school mathematics the concepts of contemporary mathematics.

Although the article does not venture to predict which path we will follow, you will want to read it for the implications found there.

"Fat Measuring Tool." *The Purdue Agriculturalist*, vol. 46, December 1954, p. 3.

Mathematics and measurement cannot be separated. With the use of many different types of measuring instruments the concept of quantity is developed. Those of you who live in the corn belt states will certainly want your mathematics students to read this short article about a new instrument measuring the thickness of fat and lean on live animals. A simple little instrument using the fact of differences in the electrical properties of fat and muscle enables one to select his kind of bacon on the hoof. It is predicted this measuring instrument will do for animal husbandry what the Babcock butterfat test did for the dairyman. Your farm boys will be much interested.—PHILIP PEAK, *Indiana University, Bloomington, Indiana.*

Stimulating interest in junior high school mathematics

ETHEL HARRIS GRUBBS, *Public Schools of the District of Columbia, Washington, D. C.*

The author offers many suggestions for maintaining interest in mathematics but remember—it is the teacher that counts.

"A JUNIOR HIGH SCHOOL TEACHER must be original to survive." This was said by one of my junior high school teachers of mathematics. Perhaps not entirely original in idea, the statement is certainly almost axiomatic. It is a truth emergent from a living experience—and we respect it as such.

The National Council of Teachers of Mathematics affords an excellent opportunity for the exchange of ideas through its annual meetings. Many of the teachers under my supervision have, from time to time, successfully used many of the ideas and materials which had been presented by other teachers at the annual meetings of the Council. Yet, the "original" teacher, the "good" teacher, must to a large degree create a good part of her own methods and materials. To maintain and extend the interest of the junior high school students—the most difficult of all school age groups—has been the main objective of many of my teachers. A few inventive plans which have worked with some of the mathematics teachers of Washington, D. C., make our contribution to the general exchange of ideas. These plans have been chosen upon the basis of unusual teaching method, teaching procedure, or outcome. I have selected only those ideas which I have not seen presented at meetings of the Council.

My illustrations are taken from our work in the following categories:

1. Community problems in arithmetic (a specific instance: taxation)
2. Informal geometry and measurement
3. Motivation for the occupational group
4. Fundamental operations and skills
5. Motivation through real problems
6. Algebra and numerical trigonometry

A teacher once told me that her pupils had more interest in the topic of taxation than she had. The District of Columbia, as you know, is the disenfranchised area of the United States. The ballot is available to the District resident only if he maintains voting residence in one of the states. Since he is taxed similar to citizens in the states (but without representation), the situation has become a major issue with many of the District people. In spite of this, many teachers do have a natural enthusiasm for teaching taxation.

My first illustration concerns the work of a class studying taxation. The unit of taxation usually follows the study of budgets. The students progress from the study of individual budgets through family budgets to municipal and federal government plans for spending income.

When I first visited this class I observed that the pupils were arranged in rows according to lot number and block number similar to the actual system of designating real estate lots in the District of Columbia. Some seats represented residences, while other seats represented apartment build-

ings, hotels, and stores. A large chart had been drawn by the students showing the various departments of the city government, as fire department, police department, public schools, courts, health department, libraries, public welfare, recreation, and the like. Pupils had consulted the telephone book for lists of all departments. Three students acted as the three commissioners. Some of the other students acted as heads of the respective departments, some acted as assessors, and one was the tax collector. They made use of current newspaper clippings. The District budget was, in actuality, being discussed at the time. The "department heads" made out their budgets and turned them in to the "commissioners" for approval. The "assessors" made a trip to the District Building where they took a look at the plat books. They had a long interview with one of the District commissioners who took time to explain to them the work of the assessors in the study of land evaluation for every foot of ground in the District of Columbia. The students returned and reported to a highly interested class. They made out tax slips for each member of the class according to his holdings. Payments were made to the student tax collector by checks made out in class. I believe that this class developed a genuine appreciation for the necessity of taxation.

While real estate taxation is only one source of tax income, I am of the opinion that with the approach through the study of real estate taxes the foundation is laid for an appreciation of income, sales, and other kinds of taxes.

During the year that sales taxes were first levied in the District of Columbia, another class, studying taxation, debated whether sales taxes are direct or indirect taxes. After an argumentative and interesting discussion the class designated one of its members to write to the United States Treasury Department. His commission was to state the problem and ask for a decision. A deputy commissioner of internal revenue wrote a page and a half in reply, explaining fully why sales tax is considered an indirect tax. I shall quote a part of that letter:

"In the field of tax law the question as to whether a tax is direct or indirect generally arises in determining whether it is in conflict with Article I, Section 9 of the Constitution of the United States which prohibits the Congress from levying a direct tax unless it is apportioned among the States according to the population. A direct tax has been defined by the Courts as including a capitation or poll tax and also as a tax imposed upon property by reason of its ownership. A tax on real estate for example, is a direct tax.

"Sales taxes, commonly referred to as excises, are indirect taxes as they are not imposed upon property as such but upon the transaction of selling property. This is true whether the tax is payable by the seller or purchaser. . . . "

The pupils enjoyed their contact with the revenue authorities. Whenever such activity can be arranged, it is usually an added incentive for studying the topic.

A teacher is reasonably sure to get the attention and interest of all pupils in the class during the study of measurement if the teacher has for demonstration purposes at least one attention-getting object. If that object is the personal property of one of the class, interest is "sure-fire." For instance, in a class which was studying the relationship between circumference and diameter of a circle, one of the boys proudly brought for the demonstration his shining new bicycle. Another boy brought his drum. Other students felt the responsibility for bringing in round objects in order that they might better participate in the study of measurement of circles. A delightful photograph of the class was taken revealing the intense interest of the participants.

Similarly a teacher brought a real electric meter to class when the group was studying how to read meters. The real

meter supplemented the diagrams which the teacher had on the blackboard and gave real meaning to a life situation.

Seventh-grade youngsters just beginning the study of informal geometry were moved to great enthusiasm by making a collection of buttons and earrings which they mounted on a large chart and classified as to straight lines, curves, circles, combinations of straight lines and curves, and various solids. Interest grew as the hunt began, for the pupils had no idea, at first, that they would find so many different shapes in buttons. A girl, in making a poster on geometric figures, knitted the figures of different shapes and placed them on a cardboard, which made a very attractive poster. In still another class, a group of boys made a small quilt, sewing together sections made of hexagons, squares, rectangles, triangles, and other figures.

Perhaps the idea of the construction of a model house by a class studying informal geometry and scale drawing is not new, but I shall describe this activity because the outgrowth of the project revealed a genuine interest in and appreciation for mathematics which persisted long after the completion of this particular class project.

A rather progressive and aggressive class enjoyed additional assignments and creative work. They began the construction of a model house, thus utilizing their knowledge of geometry and scale drawing. Since the topic of scale drawing came near the end of the semester, and because of some unforeseen interruptions, the semester came to a close before the project was finished. The teacher promised the pupils that she would permit them to complete the project the following year. Fortunately, the teacher taught the same pupils the next term.

In order not to interfere with the regular work of the new grade, the pupils and teacher took extra time during afternoon periods to work on their house. The "builders" made an extraordinary picture—stretched out upon the newspaper-covered classroom floor, besieged by paste, geometrical models, and measuring instruments. Each section of the construction was taken care of by a particular team. Woe betide the team whose section did not measure up to requirements! After a hilarious two weeks the house was completed—and a triumph of architecture it was.

At the end of the following year this class was among the candidates for graduation. Muriel, one of the members, was not among the first three honor candidates for graduation, and, therefore, would not ordinarily have an opportunity to give a graduation address. However, she begged so hard for a chance to give a graduation speech, that, in view of her good scholarship and her excellence in dramatics, she was promised a hearing if she developed well an excellent idea. Muriel went to her mathematics teacher with this plan: using as concrete material the house the class had built, she would create a sketch and call it "Building My Life at Terrell." Muriel supplied all the facts, while the teacher prepared descriptive blank verse. Other members of the class contributed through the construction of additional visual materials, for they knew that Muriel was representing them. Muriel's graduation speech was a great success. She prepared and arranged figures in the shape of a tower. This visual aid made a lasting impression on the minds of the audience.

Several years ago considerable attention was given by school officers and teachers to the classification and instruction of that group of pupils in each junior high school called an "occupational group." The definition of a child for placement in an occupational class as given by our first assistant superintendent at a meeting of the Board of Junior High School Principals was: "A child who gives evidence of not being successful in academic work or

becoming a skilled mechanic. That means that he does not show evidence of possible success as a tradesman or skilled worker in academic pursuit, but that he does give evidence of some ability to obtain information with regard to occupations that might lead toward future work for him and good citizenship." Criteria for identification of occupational class pupils included mental measurements, chronological age, achievement scores, teachers' judgments, unsatisfactory progress in school work. In general, the I.Q. range was from 70 to 79. Teachers were selected from the regular faculties on the basis of their sympathetic understanding of and appreciation for the problems of the children. The heads of departments, in cooperation with the principals, organized curricula adjusted to the abilities and levels of achievement of the pupils placed in these classes.

In one school 18 girls in an occupational group somehow learned of their classification which they bitterly resented. They were determined to be unco-operative in spite of the teacher's efforts to assure them they held an enviable position because of their small class. It became necessary for the teacher to use a great many techniques and devices in order to capture their interest. Geometry of form was taught through the construction of "touches of white" for the neck. Pictures of collars and jabots were brought into class. A diagram of one of the pictures was placed on the blackboard each day. There were discussions of the shapes used in making the jabots and collars, samples of materials that might be used, and suggestions as to how they might be made attractive. The members of the class decided that they would like to make two articles in the mathematics class. They used white paper and planned to ask the domestic art teacher to allow them to reproduce the paper models in cloth later if they turned out well. Through their construction work the pupils learned the properties of the following geometric figures: the square, rectangle, circle, semi-circle, and the quadrant. They learned to use and understand the relationships of radius, diameter, and circumference. They found it necessary to use horizontal, vertical, and oblique lines. The perimeters of the square and rectangle were studied when the border designs were made for the square and rectangular collars. Pupils learned to use the ruler and compass. They learned about fractional parts through their use of the fractional parts of the inch. A few students found it necessary to develop skill in cutting before they could work on their project. When the "touches of white" were completed, the pupils were very happy to be allowed to bring members of other classes into the room to see the work on display. After this, the teacher had no difficulty in maintaining their interest.

Drill can be dull and often is. We often wonder why, after many years of teaching fundamental operations, some of our best pupils do not measure up to standards. We all know that one of the criteria for effective drill is interest.

A mathematical baseball game which differs in some respects from real baseball, was devised by a teacher as an innovation to get and maintain interest in drill work in connection with various computational skills. The game includes two teams with eight or more pupils on each team. Players are usually selected by the pupils. Material used may be any sort of combinations which may be answered quickly, such as: Add 7, 6, 9; combine $13-5+2$; $+7x-9x$; 80% of 35; etc. The game begins with easy combinations and gets more difficult. A space in the classroom is planned to correspond with a baseball diamond. The pupils take their respective places on the diamond. The first baseman and the catcher on each team must be the strongest, i.e., the quickest and most apt members of the team. The pitcher must be clever at picking the right combinations for certain batters. When the

pitcher throws (gives) a combination to the batter, and the batter does not give the correct answer before the first baseman, he is out. Pupils simply move around to bases until it is time to come into "home." Then the pitcher throws another combination to the man on third base. If he is quicker than the catcher, he may come into "home," if not, he is out. A scoreboard is used. This game has also been used successfully in interclass competition, on assembly programs, and at the end of any unit. Pupils work hard during the study of a unit in order to be prepared for the game which is to follow.

Interest aroused by real problems is one of the best forms of motivation.

I should like to tell you something about the work of a teacher with a group of ninth-grade students who would ordinarily have been taking general business and commercial arithmetic. The project took the place of both of these courses. Commercial arithmetic and general business courses do not come within the purview of the mathematics department in our school system, but I cite this example for two reasons: First, the teacher chosen for the project was one of the teachers in the mathematics department, and second, although I am not enthusiastic about incidental learning of mathematics through projects, when the project is in the hands of a good mathematics teacher who recognizes the mathematical implications inherent in such a situation, the activity then offers an excellent opportunity for the acquisition of mathematical knowledge in an orderly manner.[1]

All activities of this group grew out of actual problems of the school cafeteria. Each pupil worked on a specific problem and each was responsible for all records pertaining to a particular company with which the cafeteria did business. A few of the activities involving mathematics were: Checking and recording daily invoices, preparation of vouchers for filing, determining selling prices, evaluating monthly invoices, analysis of Workmen's Compensation insurance, wise buying—as for example, whether it was more economical to buy No. 1 or No. 2 cans of a product; assisting cafeteria workers in the preparation of their personal income tax returns; preparing graphs of all types showing changes and comparisons. In assisting with the analysis of profit and loss statements, students spent long, interesting periods computing percentages. Their frequent use of all types of percentage problems resulted in their ability to attack any percentage problem with complete ease and understanding.

For motivation of the motion problems in algebra, particularly those involving wind and water currents, and for illustrating the fact that a plane flying with the wind has a rate of $x+y$ and against the wind a rate of $x-y$ where x equals the rate of the plane and y equals the rate of the wind, a toy plane was suspended by a cord and a wind created by a fan. This illustrated the fact that the wind gave the plane an additional rate when it was traveling in the same direction with the wind. A toy boat was placed in a large bread pan into which water flowed creating a current. The effect of trying to go against the wind or water was experienced when the pupils tried to push the plane toward the wind with one finger or to push the boat upstream with one finger.

To stimulate interest in a review of factoring and special products, one teacher likened her pupils to detectives who must first find the clues (recognize the cases in the example). Then they must use the clues to solve the particular situation according to accepted methods.

I am sure that almost every teacher of algebra who has taught for any length of

[1] Information about this group of students was published in an article in the December 1949 issue of the *Balance Sheet*, under the title, "An Experience in Pupil-Teacher Planning," by Gwendolyn Holland.

time has at some time had one or more pupils construct transits and other homemade measuring instruments in order to arouse interest in numerical trigonometry. Also many teachers have their pupils, who have a flair for drawing, illustrate their problems by drawings. About three years ago, one of our teachers varied her techniques by suggesting that each one of the class should construct something during the semester which would illustrate the impressions that his problem work had made upon him, each construction to be accompanied by a related problem and its solution. Some students illustrated work problems, using toy figures to represent people and materials appropriate to the work indicated by the problem, as toy brooms, picks, shovels, coal, mounds of dirt with indentations to represent ditches, etc. Some illustrated problems of the lever by constructing teeter boards, placing toy people on the ends. Other types were illustrated. One outstanding construction was made by a boy who before that time had been barely getting his work and had not shown any interest. His illustration of a problem involving numerical trigonometry turned out to be the most painstakingly and thoughtfully constructed. His model consisted of a large board with a section painted blue to indicate the ocean, a toy boat on the water, and at one corner he built a lighthouse of plaster of paris with glass panels at the tip to enclose the observation tower. He wired the lighthouse for electricity and placed in the tower a tiny electric light bulb that actually worked. While working so enthusiastically on his project, this boy became more interested in the regular work of the class.

In conclusion, it should be kept in mind that in using any idea for stimulating interest the personality of the teacher plays the most important part in the achievement of a desirable rapport between the teacher and pupils in the classroom. Creativeness combined with a sense of humor, and a deep abiding faith in the ultimate worth of each individual are indispensable "musts."

Speaking of humor and rapport, I should like to tell this anecdote concerning one of my teachers. Once each term she designates a day as "Teach the Teacher Day." She encourages pupils to give criticisms concerning her teaching methods and procedures in order that she might study the criticisms and probably find some suggestions which will help her plan for the next term. The criticisms are not to be signed by the pupils, and indeed she suggests that maybe some of the pupils might not like to write but would prefer to tell their impressions by drawing. Here is what one boy wrote: "You are hard on the children and that is the way it should be. I do not like home work on Mondays because we skip class the next day and forget about it. Picture over." On the back of the sheet he had drawn a cartoon of a teacher wearing eyeglasses and a long robe, hands folded together in front, wings attached to her back, a halo over her head, and caricatures of imps before her. Across the top of the page he had printed: "IT'S JUST LIKE AN ANGEL TEACHING A BUNCH OF DEVILS."

"The most difficult thing for a teacher is to recollect how much it cost him to learn, and to accommodate his instruction to the apprehension of the uninformed."—*Thomas Young*, Natural Philosopher by Alex Wood, *Cambridge University Press, 1954*.

Edited by Phillip S. Jones, University of Michigan, Ann Arbor, Michigan

Tangible arithmetic IV: finger reckoning and other devices

by Phillip S. Jones

The finger symbols shown in our Figure 1 were once used to communicate at international fairs and as an aid in remembering numbers when using an abacus. This picture is from Philippi Calandri's *Arithmetice* published in Florence in 1518. The first edition of this book is an incunabulum which means that it was printed during the "cradle" days of printing, which are taken to be its first fifty years, 1450–1499. The first edition was printed in 1491, but finger symbols existed in Greece as early as the fifth century B.C. and it has been stated that some also existed in China as early as the time of Confucius (551–478 B.C.).[1] Smith notes that finger symbols even appear in art and literature. For example, he cites a passage from Juvenal who said "Happy is he who . . . numbers his years upon his right hand," referring to the fact that the numbers less than one hundred were on the left hand and greater than one hundred on the right.[2]

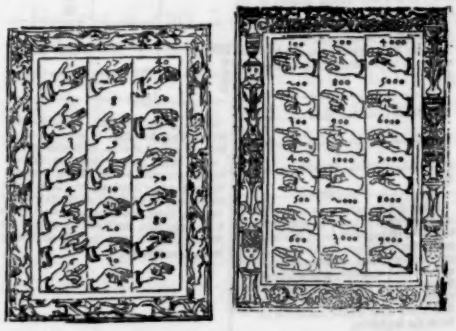


Figure 1

There were even devices for using fingers to operate with numbers as well as to record them. For example, a device for multiplying integers between 5 and 9 was to hold up fingers on one hand to show by how much the multiplier exceeded 5 and on the other hand to show similarly the amount by which the multiplicand exceeded 5. The product of the original numbers was the number whose tens digit was given by the sum of the fingers held up and whose units digit was the product of the fingers turned down on the two hands. For example 7×9 would lead to 2 and 4 fingers up and 3 and 1 fingers down. This would give the product 63. For this process one needed to know the multiplication tables only up to 5×5. The explanation of this device is enrichment material for algebra as well as for arithmetic since it hinges on the identity

$$ab = 10[(a-5)+(b-5)] + [(10-a)(10-b)].$$

Other devices for recording numbers which should be included in an exhibit, display, report, or assembly program on tangible arithmetic are the *quipu*[3] used

[1] D. E. Smith, *History of Mathematics*, Vol. II (Boston: Ginn and Co.), p. 197. Chapter III, pages 156–207, titled "mechanical aids to computation" discusses many tangible computing devices.

[2] Smith, *loc. cit.*

[3] Smith, *op. cit.*, p. 195.

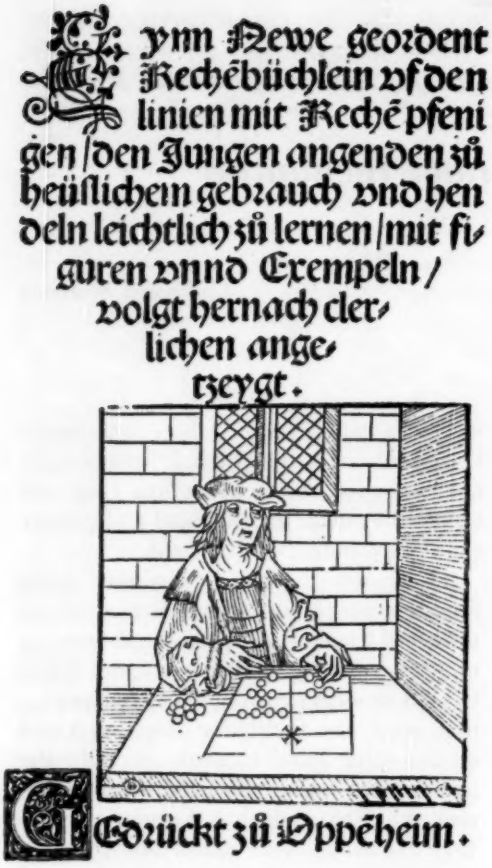


Figure 2

by Peruvian natives before the time of the Spanish conquerors, and *tally sticks*[4] used in medieval England. Encyclopedias will give some help on these items as well as the references cited in the notes.

Among computing devices the *abacus*[5] is so well known that we will only mention it here with a few references, and the *counting board* has been so recently discussed in this journal[6] that we will only add Figures 2 and 3 to the earlier data. Figure 2 is the title page of a German book on "Rechnenpfenigen" by J. Köbel published in 1514. It shows a merchant at his counting board with the lines drawn or cut into it. Note the title which reads "A new kind of little reckoning book (telling of reckoning) on lines with reckoning pennies." These "pennies" were often little stones from whose Latin name, "calculi," we get our words "calculate" and "calculus" just as our word "counter" for a table or bench in a store comes from the medieval merchant's counting board.

Figure 3 shows an explanation of the multiplication of 1542×365 on a counting board from the most famous early English arithmetic. This is Robert Recorde's *The Grounde of Arts* first published about 1542. Our picture is from the 1654 edition and shows one of the many interesting features of Recorde's works; namely, they were written in the form of a dialogue between the "Master" and his "Student." The arithmetics we have pictured display three significant common and related char-

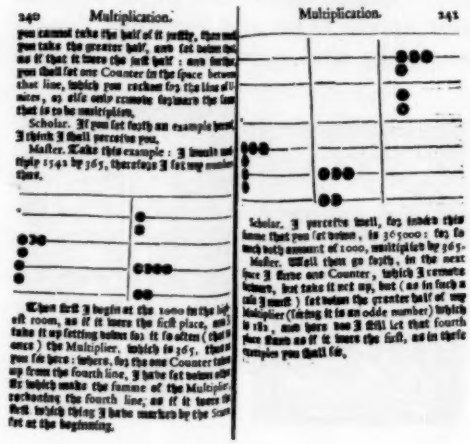


Figure 3

[4] Smith, *op. cit.*, p. 192. Vera Sanford, "Art of Reckoning: I. Tallies," THE MATHEMATICS TEACHER, Vol. XLIII (October 1950), p. 292.

[5] Smith, *op. cit.*, pp. 156–192. Vera Sanford and Yen Yi-Yun, "The Chinese Abacus" THE MATHEMATICS TEACHER, Vol. XLIII (December 1950), p. 402 ff. T. Kojima, *The Japanese Abacus, Its Use and Theory* (Rutland, Vt: 1954). Jerry Adler, "So You Think You Can Count!" *Mathematics Magazine*, Vol. 28 (Nov.–Dec. 1954), pp. 83–86.

[6] Smith, *op. cit.*, p. 181. Vera Sanford, "Counters; Computing If You Can Count to Five," THE MATHEMATICS TEACHER, Vol. XLIII (Nov. 1950), p. 368.

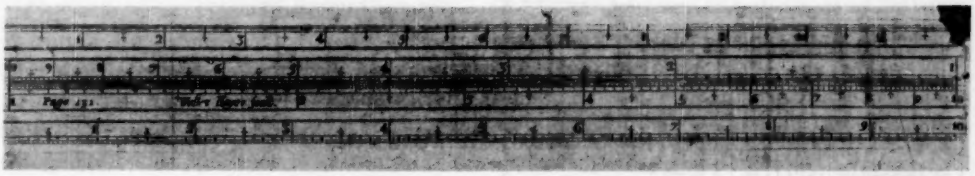


Figure 4

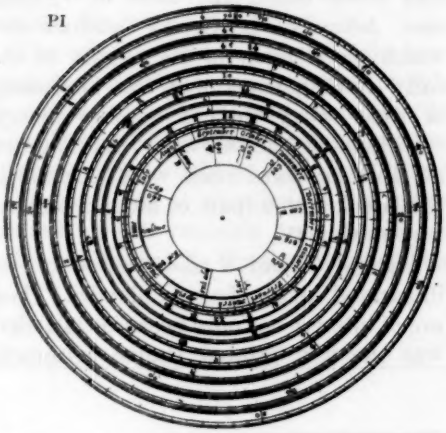


Figure 5

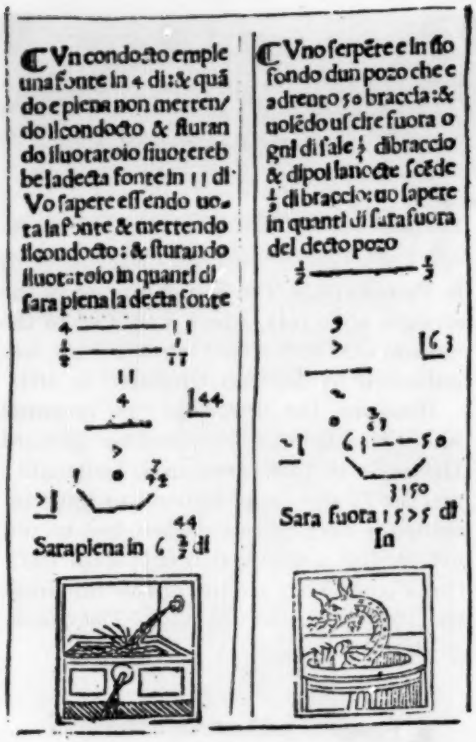
acteristics: their period, the late fifteenth and early sixteenth century; their use of the common or "vulgar" languages rather than Latin, the scholarly international language; and their concern with the computations of commerce.

Figures 4 and 5 relate to two other well-known and recently discussed computing devices, *Gunter's scale* and the *slide rule*.[7] Figure 4 shows the scale as it appeared in the sixth (1698) edition of William Leybourn's *The Line of Proportion or Numbers, Commonly Called Gunters Line, Made Easie.* . . . Although Edmund Gunter was an Englishman, he published his first discussion of his scale in French in Paris in 1624. The first discussion of the straight slide rule, which would seem the natural next step after Gunter's scale, was published by William Oughtred in 1633.

However, the first slide rule invented was a circular rule described by Richard Delamain in 1630. Oughtred, Delamain's teacher at one time, claimed to have invented a circular rule earlier, but he did not publish a description of it until 1632. Our Figure 5 is a picture of this rule from the 1660 edition of his book, *The Circles of Proportion.*

Figure 6

[7] Phillip Jones, "The Oldest American Slide Rule," The Mathematics Teacher, Vol. XLVI (Nov. 1953), p. 501. Florian Cajori, "On the History of Gunter's Scale and the Slide Rule During the Seventeenth Century," *University of California Publications in Mathematics*, Vol. 1 (Feb. 17, 1920), pp. 187–209. Smith, *op. cit.*


Vn condocto emple una fonte in 4 di: & quã do e piena non mettendo il condocto & sturando il suo tarolo fuori ereb be la decta fonte in 11 di Vo sapere essendo uota la fonte & mettendo il condocto: & sturando il suo tarolo in quanti di sara piena la decta fonte
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Vno serpẽte e in fo fondo dun pozo che e adrento 50 braccia: & uolẽdo usc fuora o gni di sale 1/2 dibraccio & dipoi lancte scẽde 1/3 di braccio: uo sapere in quanti di sara fuora del decto pozo
Sara fuora 1575 di
In

Figure 7

The *nomogram*[8] is a recently developed computing chart which uses logarithmic scales and could well be included in a tangible arithmetic project. Frenchman Maurice d'Ocagne began its modern phase in 1891.

Computing machines are so in the public eye and press today that it is not necessary to do more than mention them in this connection. Clippings, pictures, demonstrations of desk calculators and an outline of their historical development[9] would be part of a "tangible arithmetic" project.

[8] Florian Cajori, *A History of Mathematics* (New York: Macmillan, 1919), p. 481. Douglas P. Adams, "The Preparation and Use of Nomographic Charts in High School Mathematics," *Eighteenth Yearbook*, National Council of Teachers of Mathematics, pp. 164–181.

[9] Smith, *op. cit.*, p. 202. Phillip S. Jones, "The Binary System," THE MATHEMATICS TEACHER, Vol. XLVI (Dec. 1953), p. 575. Philip and Emily Morrison, "The Strange Life of Babbage," *Scientific American*, Vol. 186 (April 1952), pp. 66–73.

Calandri's Arithmetic:

The book from which our Figure 1 came is such a fascinating little work that we include a few more pictures from it. Figure 6 is its title page. This illustrates a false notion of that day, that Pythagoras introduced the Hindu-Arabic numerals and their arithmetic into Europe. The square form of the multiplication table was also often called the table of Pythagoras. Actually, the Greek "arithmetica" which owes much of its origins to the early Pythagoreans was the beginnings of what today we call number theory. "Arithmetica" neither used Hindu numerals nor concerned itself with "logistica," the Greek counterpart of modern elementary arithmetic.

Figures 7 and 8 show some of the illustrated pages from Calandri. These are of interest not only because Calandri's was the first printed Italian arithmetic

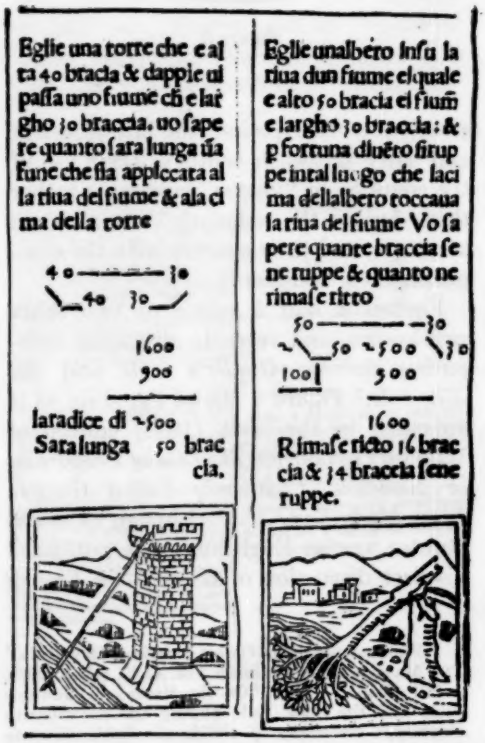

Eglie una torre che e al ta 40 bracia & dapple ui passa uno fiume ch̃ e lar gho 30 braccia. uo sape re quanto sara lunga ũa fune che sia applecata al la riua del fiume & ala ci ma della torre
1600
900
laradice di 900
Sara lunga 50 brac cia.
Eglie unalbero insu la riua dun fiume elquale e alto 50 bracia el fiũ e largho 30 braccia: & p fortuna diuẽto sirup pe intral luogo che laci ma dellalbero toccaua la riua del fiume Vo sa pere quante braccia se ne ruppe & quanto ne rimase ritto
1600
Rimase ricto 16 brac cia & 34 braccia sene ruppe.

Figure 8

with illustrations accompanying the problems, but also because the problems themselves are so similar to current ones that one can figure them out from the pictures, the numbers, and a few hints. Try them on some of your students. Figure 7 shows a serpent who each day climbs 1/7 "bracchia" (a unit of measure) out of a well only to slip back 1/9 bracchia. If the well is 50 bracchia deep how long does it take him to climb out?

The other picture in Figure 7 asks how long does it take to fill a trough or tub if water enters at a rate such that it would be full in 4 days if it weren't leaking at a rate which would empty it in 11 days?

Figure 8 asks at what height above the ground the 50 bracchia tree should be cut in order that its tip should touch the bank of the river at a distance of 30 bracchia as shown. Clearly this involves the Pythagorean theorem, but one might challenge students to set up algebraic expressions which would explain the process by which Calandri arrived at the correct answer, 16.

Also to be found in this book are "work problems," a lion, leopard, and wolf who eat at different rates, and ants who travel toward each other at different rates. Problems on the altitude of an equilateral triangle and circumference and area of a circle are also included in this book which was the first to give our modern form of long division and to omit the old "galley" type of division.[10]

[10] D. E. Smith, *Rara Arithmetica* (Boston: Ginn and Co., 1908), p. 47 ff.

Have you read?

SALYERS, GARY D. "The Number System of the Mayas." *Mathematics Magazine*, Vol. 28, September–October 1954, pp. 44–48.

Usually one thinks of the development of our number system as being wholly a product of the far east. However, Mr. Salyers' article points out that the western hemisphere may have contributed a great deal to our present concepts. The Mayan civilization was highly advanced and had a number system with a base of 20. Their symbols were meaningful with $\cdot = 1$, $\cdot\cdot = 2$, $\cdot\cdot\cdot = 3$, $\cdot\cdot\cdot\cdot = 4$, $- = 5$, $\doteq = 6$, etc., adding lines and dots to 19. Above this they used vertical place value rather than horizontal; for example, $21 = \dot{:}$, $23 = \dot{:}\cdot\cdot$. Zero was not a part of the system until about 500 A.D. and it seems to have been discovered independent of the Hindus. Therefore, it was necessary to write 20 as $\cdot\,\dot{}$, showing that the bottom space was blank. In other numbers a space was left; for example, $7202 = \therefore$ with two spaces between top and bottom row. 7202 shows an interesting variation in that the third place was 360 rather than 400. Just why this was true is unknown, but since the number system and the calendar were closely allied it was probably due to the solar year.

The Mayas had lucky numbers, 9 and 13. They also used pictures with the skull representing 10. There were many other variations; but you will want to read this as well as recommend it to your students, not only for its interest but also for the light it sheds on number systems as such.

HILLS, E. JUSTIN. "Alligation—Its Meaning and Use." *Mathematics Magazine*, November-December 1953, pp. 85–89.

All of you occasionally find yourselves teaching mixture problems. Only rarely do you use a problem in which a complex combination of elements is included. This article provides an excellent source of information for the better student in your class who wants to go beyond the simple linear system solution. Alligation, the Rule of Mixture, is of two types—that of medial or the mean value, and the alternate. It is the latter that Mr. Hills discusses. He presents three alternate procedures which are not new but seldom appear in print today. The three are: first, "percents differences," second, "plus and minus differences," and the third, the "link" method. This latter method is adaptable to many mixture situations where the equation method does not apply. I am sure your better students will enjoy this when studying mixture problems.—PHILIP PEAK, *Indiana University, Bloomington, Indiana.*

• MATHEMATICAL MISCELLANEA

Edited by Paul C. Clifford, State Teachers College, Montclair, New Jersey, and Adrian Struyk, Clifton High School, Clifton, New Jersey

An integraph

by George Kays, State Teachers College, Montclair, New Jersey

Finding an area enclosed by an irregular boundary is a problem that is quite often difficult or tedious when solved by the routine formal methods of the integral calculus. The device discussed here is a simplification of a type of instrument used to solve problems in integral calculus. Figure 1 illustrates such a problem, that of finding the area enclosed by the x-axis, the lines $x=a$, $x=b$, and the curve of the (unknown) function $y=F'(x)$.

If the function $F(x)$, an integral function of $F'(x)$, can be identified (or graphed), the measure of the required area would be $F(b)-F(a)$. $F(x)$ is, of course, any function which is continuous in the interval considered and has the slope of its graph numerically equal to the ordinate of $F'(x)$ for each value of x in that interval.

As an example consider the graph of the function shown in Figure 2. Note that the value of this $F'(x)$ is 1 for $x=0$, and thus the slope of $F(x)$ is 1 for $x=0$. The curve of $F(x)$ can then be approximated near $x=0$ by any one of the lines with slope 1 drawn through the points P, Q, R, S. Similarly, the value of $F'(1)$ is 2,

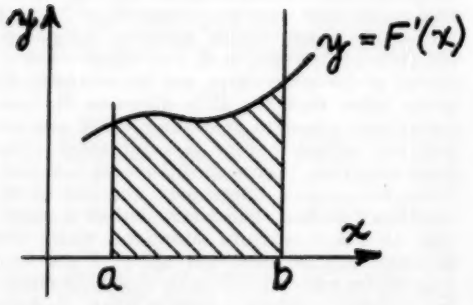


Figure 1

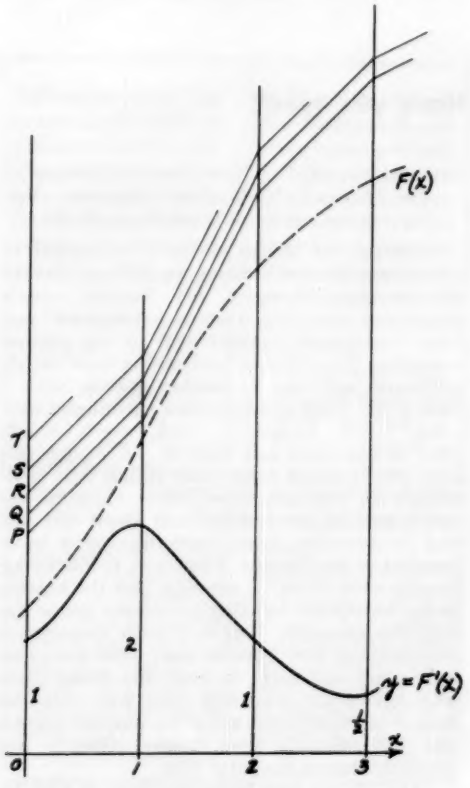
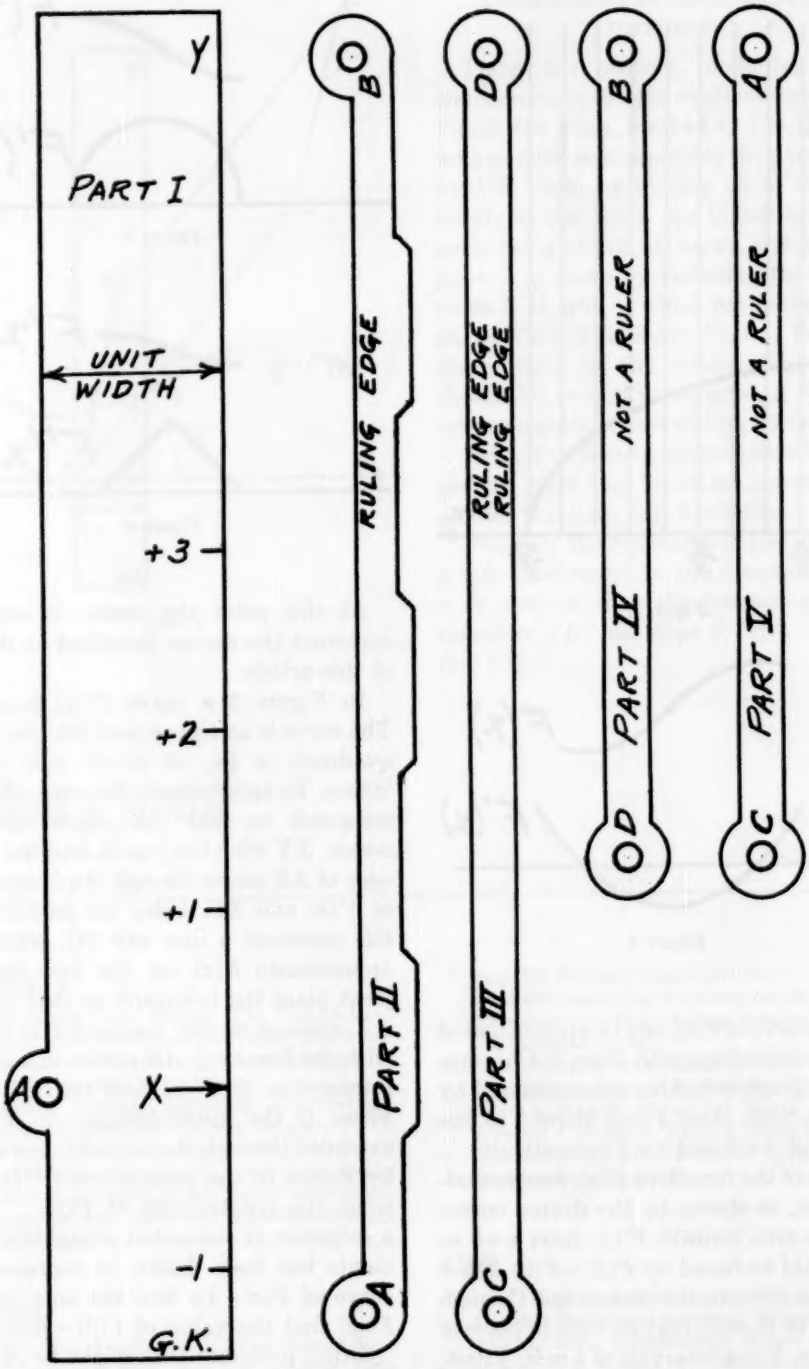


Figure 2



Pattern for integraph

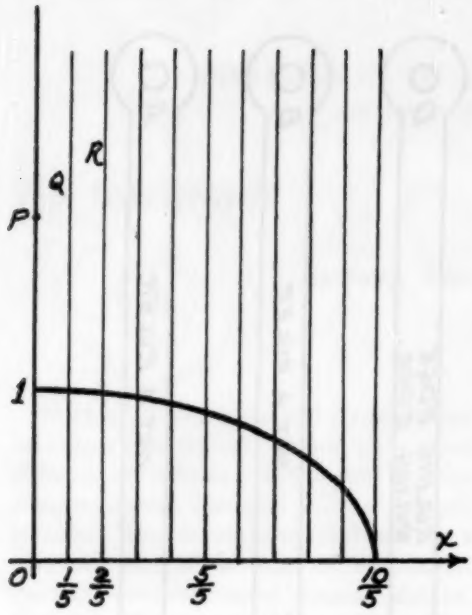


Figure 3

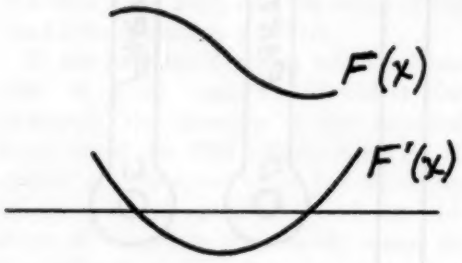


Figure 4

and the curve of $F(x)$ can be approximated by any of the lines with slope 2. Continuing, the graph is further approximated by the lines with slope 1 and slope $\frac{1}{2}$ in the vicinity of $x=2$ and $x=3$ respectively.

If one of the functions $F(x)$ were actually known, as shown by the dotted curve, then the area beneath $F'(x)$ from $x=0$ to $x=3$ could be found by $F(3)-F(0)$. Since this is not known, the broken line through Q (or P, or R, etc.) may be used in place of the curve. Using intervals of $\frac{1}{2}$ unit, $\frac{1}{4}$ unit, etc., will increase the accuracy of the approximation to desired precision.

Figure 5

Figure 6

At this point the reader is urged to construct the device described at the end of this article.

In Figure 3 a curve $F'(x)$ is shown. The curve is an ellipse, and the area of the quadrant is $\frac{1}{2}\pi$, or about 1.57 square inches. To approximate this area place the integraph so that AX aligns with the x-axis, XY with the y-axis, and the ruling edge of AB passes through the intersection of $F'(x)$ and XY. Using the parallel ruler CD construct a line, say PQ, which will approximate $F(x)$ for the first interval. Next place the integraph so that the line AX remains on the x-axis, XY is aligned with the line $x=\frac{1}{2}$, AB passes through the intersection of $F'(x)$ and the line $x=\frac{1}{2}$. From Q the approximation to $F(x)$ is extended through the second interval to R by means of the parallel ruler CD. Continue the construction of $PQR\ldots$ until a sequence of connected straight line segments has been drawn to represent the curve of $F(x)$. To find the area beneath $F'(x)$ find the value of $F(2)-F(0)$.

Other problems that might be of interest to the reader are illustrated in Figures 4, 5, 6.

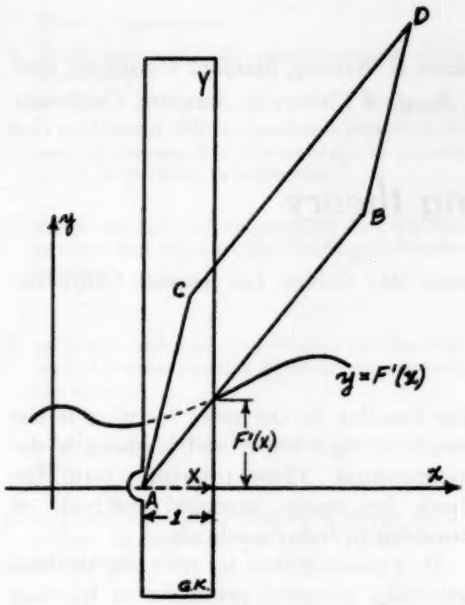


Figure 7

DIRECTIONS FOR ASSEMBLING THE INTEGRAPH

Trace the pattern, including labels, transferring it to light cardboard or O'Tag. Punch the holes, marked O. Use an eyelet set and eyelets to assemble the parts. Each joint is made by placing all of the same letters at one joint, the higher numbered parts being above the lower. For instance, joint *A* is made by fastening the ends of parts I, II, and V which are marked with an *A*. Part II is above Part I, Part V is above both of the others. The eyelets should be carefully set since, if they are set too tightly, there will be no motion.

Note: For some problems an additional parallel ruler may be added above *CD* for greater mobility and flexibility.

Proof: If the integraph is placed on the graph as directed in the discussion, then it is obvious that the pointer *AB* (and therefore *CD*) has slope $F'(x)/1$, or $F'(x)$ (see Fig. 7).

What's new?

BOOKLETS

It Takes More Than Brains, The Connecticut Mutual Life Insurance Company, Hartford, Connecticut. Illustrated booklet dealing with college expenses; 22 pp.; free.

Mathematics, College Entrance Examination Board, Educational Testing Service, P. O. Box 592, Princeton, New Jersey, or P. O. Box 27896, Los Angeles 27, California. Booklet describing the College Board tests in intermediate and advanced mathematics; 28 pp.; 50¢ each.

DEVICES

Perimeter Area Board (#763), Ideal School Supply Company, 8326 Birkhoff Avenue, Chicago 20, Illinois. Cork-faced board, 10″×20″, with 20 large pins, 6 rubber bands, and one 20″ measuring tape; 8-page instruction manual included; $2.50.

Spinner Fraction Pie Game (N248), Creative Playthings, Inc., 5 University Place, New York 3, New York, or 316 North Michigan Avenue, Chicago 1, Illinois. Game composed of wooden spinner, 6 rubber pies cut into fractional parts; $2.95.

FILM

Story of Weights and Measures, Coronet Films, Coronet Building, Chicago 1, Illinois. 16 mm. film; 11 min.; collaborator, Foster E. Grossnickle; B & W, $55.00; Color, $110.00.

Edited by Lucien B. Kinney, Stanford University, and Dan T. Dawson, Stanford University, Stanford, California

Textbook series and learning theory

by G. W. Brown, Los Angeles State College, Los Angeles, California

Do textbook series in arithmetic develop the mathematical concepts and skills in accordance with current learning theory? This is an important question for the junior high school mathematics teacher. The answer determines the extent to which the teacher may rely on the textbook for guidance in the learning sequence. Since, for many students, mathematics instruction ceases at ninth grade, mathematical strands considered by teachers and textbook writers to be important must be brought to a culminating point by that time.

Attention to a systematic learning sequence is one aspect of meaningful teaching in mathematics. Granted that there is not complete consensus among psychologists and educators as to how children learn; yet widespread agreement can be found on several principles from the psychology of learning. This agreement is recognized by Brueckner and Grossnickle,[1] for example, when they list, under the heading "Principles of Learning," thirteen statements which they suggest as a summary of learning hypotheses. Other authors[2] propose similar lists, which include such statements as: Readiness is a factor in learning; learning proceeds from the familiar to the new; learning is the result of experience; and learning is developmental. These principles form the basis for many accepted methods of teaching in today's schools.

It is possible then by referring to these generally accepted principles of learning to devise a pattern or sequence which constitutes an efficient guide in teaching mathematics.

That such a systematic learning pattern is valid in arithmetic instruction is suggested by the authors of *The Fiftieth Yearbook of the National Society for the Study of Education*, when they precede their series of learning steps in arithmetic by saying: "Learning number consists in an orderly series of experiences which begins with concrete objects and progresses toward abstractions. There are different stages, levels, or steps which may be identified in the learning process in arithmetic."[3] The "flow chart" (on the next page) described by Kinney and Purdy serves as a well-known example of such a pattern.

Arithmetic textbooks generally develop specific ideas over short periods in a manner consistent with current learning theory. The patterns of presentation (such as the flow chart), however, have impli-

[1] Brueckner, L. J., and Grossnickle, F. E., *Making Arithmetic Meaningful* (Philadelphia: J. C. Winston Co., 1953), pp. 123–124.

[2] For example, see Spencer, P. L., and Brydegaard, M., *Building Mathematical Concepts in the Elementary School* (New York: Holt & Co., 1952), pp. 25–36.

[3] "The Teaching of Arithmetic," *The Fiftieth Yearbook of the National Society for the Study of Education*, Part II (Chicago: Univ. of Chicago Press, 1951), p. 156.

DEVELOPMENT OF A MATHEMATICAL CONCEPT OR PROCESS

Order of experiences	*The teacher's responsibility*
1. Pupil, equipped with certain concepts, skills, and interests	To study each pupil and determine his readiness for new experiences and concepts.
2. is confronted with a significant situation requiring the use of a new concept or process which he is ready to improvise,	To arouse interest in a problem situation, important to the pupil, and direct his efforts in devising methods to master it.
3. which concepts or processes are then utilized in other and different significant problem situations,	To provide a variety of interesting and important situations having the process, or concept as a common element, with focus of attention on the situation.
4. until his confidence in and need for the concept or process justifies its study, as to nature and value,	To shift attention to the concept or process, studying its nature, relation to the number system, and importance.
5. acquiring skill and accuracy in its manipulation,	To be sure that understanding precedes drill, and that drill is handled effectively.
6. and then exploring and utilizing its life applications, to discover their identifying characteristics.	To provide real life experiences which call for the use of the concept or process, rather than depending on chance "transfer of training."[4]

cations for development of concepts over a period of several school years as well as for shorter periods. It is this long-range treatment of concepts that requires closer attention.

Consider, as an example, the development of formulas. As early as grade three or four, a formula such as $C=3n$ can be introduced in connection with buying stamps at the post office. The formula concept, or strand, needs to be handled thereafter in a manner which the psychologists typify as "progressive symbolization." In other words, the experience with formulas over a period of several years begins with using formulas incidentally, to assist in obtaining answers to a significant question or problem. This process corresponds to the second step of the flow chart, the first step—the readiness step—having been handled prior to the presentation of this significant question.

During the course of the following years, the use of formulas is extended to many and varied significant situations ranging from areas to interest to recipes, thus illustrating the third step in the flow chart and setting the stage for the fourth. This fourth step is reached at approximately the ninth grade. Here pupils are ready for, and are given an opportunity to focus on, the formula as a mathematical concept. This focus requires an organization of the meanings connected with formulas, generalizing as to mathematical operations involved with formulas and practice in their manipulation and use. This practice represents the fifth step in the flow chart. As a final step, then, practice is provided in identifying situations in which formulas are appropriate. A wide range of applications is required, varying in areas and in degrees of difficulty, to insure adequate transfer to life situations.

Do textbook series consistently handle mathematical processes as strands in this manner? Several popularly used series of textbooks were examined for grades seven, eight, and nine to determine how some of the strands were developed during

[4] Kinney, Lucien B., and Purdy, C. R., *Teaching Mathematics in the Secondary School* (New York: Rinehart & Co., Inc., 1952), p 54.

the junior high school period. The strands were divided into three categories: (1) *Measurement*—which included such things as volume, weight, and circle; (2) *Expression of ideas*—which included graphs, formulas, tables, percentage, number meanings; and (3) *Fields of application*—which included the home, business, immediate age interests, conservation, etc.

The treatment was very uneven. It appeared that several topics, notably graphs and formulas, were brought to a point of culmination during the junior high school years, which to some extent was consistent with current learning theory. On the other hand, different means of expressing ideas, such as tables, maps, and scale drawings, were very inadequately handled from this point of view. Percentage, as a topic, is studied in all three years. However, there was little indication of a co-ordinated progression from a concrete level in earlier grades to an abstract presentation in the ninth grade. This is not to say that ninth-grade problems were not more difficult than the seventh-grade problems. In many cases, however, the principal difference in the presentation for the three grades was the social setting of the units.

In the measurement category, linear, area, and volume measurement were dwelt upon at length at all three grade levels, whereas liquid and dry measure, time measures, and circular measure received little attention. In none of them was the "... orderly series of experience which begins with concrete objects and progresses toward abstraction"—which is advocated by the National Society for the Study of Education's *Fiftieth Yearbook*[5]—clearly in evidence.

[5] "The Teaching of Arithmetic," *op. cit.*

Why is this of special concern to the mathematics teacher in the junior high school? Actually, the teacher is directly concerned. The mathematics program in the junior high school has the unique function of abstraction and symbolization. Few, if any, new processes are introduced at this level; and whereas the emphasis now is typically on applications, that emphasis might be more effective according to current learning theory if shifted to the abstraction and symbolization of the mathematics strands. The abstractions, as a final activity, should be largely those of identifying the kind of situation to which the process applies. Since the situations themselves will change, the emphasis here is on the *kind* of situation.

A second concern of the teacher is that if the textbook does not adequately treat the strand, then the teacher must. For teachers this means engaging in considerably more longitudinal planning than is usually done. Further, it will require leadership on the part of teachers, heads of departments and supervisors to initiate this planning and to carry it to completion.

Growing out of this longitudinal planning could very well be the desperately needed type of research which represents a third concern of the junior high teacher. Research is needed to reveal stages of abstraction appropriate for the various grade levels and experiences suitable to each stage. This is the type of research to which classroom teachers, utilizing proper evaluation programs, can contribute by planning, building, and testing units which provide for experiences at various levels of abstraction and degrees of symbolization.

• MEMORABILIA MATHEMATICA

Edited by William L. Schaaf, Department of Education, Brooklyn College, Brooklyn, New York

Some reflections on the printing of mathematics

This department, for a variety of reasons, has always been deeply interested in the making of mathematical books. It was with more than a little delight, therefore, that we came upon a fascinating book[1] devoted to the art of printing mathematics. Until about a quarter of a century ago, the great bulk of mathematical composition was set by hand. Since then, the prolific volume of contemporary writing and an ever-increasing scientific and mathematical reading public have compelled the printing trade to utilize the resources of the machine and of the machine compositor for the exacting demands of mathematical printing. And so, with the reader's indulgence, and with the kind permission of the Oxford University Press, let us look at a few illuminating excerpts from this timely monograph.

First, some general observations:

The production of a piece of printed mathematics requires for its successful completion the cooperation of three parties—the writer, the printer, and the reader. Because of the abstract nature of mathematical thought and of its essential use of symbolism, the writer has always a difficult task in carrying over his meaning to the reader quickly and without uncertainty. The printer, as a middleman, has the double duty of catching the writer's meaning and of passing it on unimpaired to the reader.

The setting of mathematics is a rare and expensive skill not readily acquired, and existing facilities often have difficulty in keeping pace with the rapidly growing output of mathematical writing. For this reason there may be a long and vexatious delay between the writing and the publication of a mathematical work.

A greater and more abiding anxiety is that of the steeply rising costs of mathematical printing. Learned bodies charged with the maintenance of mathematical periodicals are faced with difficult economic problems: are they to raise prices at each increase of costs or must they steadily cut down their acceptance of material? Similar questions confront the publishers of mathematical books.

A few points of interest to the mathematical writer:

It is a sobering experience for an author to contemplate the chain of processes that he sets in motion when he sends a manuscript for publication.

Authors should be more aware of the nature of composed type:

The basis of composition, and especially of mechanical composition, is the formation of lines of type of uniform depth. The printer, like the determinist in the philosophical limerick, is

> *"A creature that moves*
> *In determinate grooves:*
> *In fact not a bus but a tram."*

In fact, in "Monotype," with its precise units of type-width as well as the point units of body-depth, we can regard composition as if it were set out on graph paper. On the other hand the mathematician's pen is free to roam, enlarging or compressing his script, filling blank space with symbols, constructing marvels of formulae —even, perhaps, ringing a character or inserting one character within another in a way that cannot easily be directly imitated with metal type.

One of the many problems involved in the printing of mathematics is the use of characters which can be set by "linear workings" (laterally) on a machine versus those which are set vertically as pieces of type inserted by hand. We read:

It is essential to mathematical notation to be able to show association of symbols by symbols suitably "embellished": thus with x we can associate x', x_1, $\bar{x}$, $\dot{x}$. Here again lateral additions as in x', x_1 are much to be preferred to vertical additions as in $\bar{x}$, $\dot{x}$. The prime and the suffix are put in directly from the keyboard. . . . Vertical

[1] *The Printing of Mathematics*, T. W. Chaundy, P. R. Barrett, and Charles Batey (New York: Oxford University Press, 1954), 105 pp.

additions, on the other hand, must be inserted on assembly as pieces of type that must be supported. . . . The author who is accustomed to produce the text of his work on a typewriter, afterwards inserting symbols and formulae with the pen, will be aware of the care and delicacy needed to avoid errors and omissions at this second stage.

Nor are the concerns and convenience of the reader overlooked in this stimulating monograph:

. . . the author therefore has the obligation to see not only that the presentation of his mathematics is complete and impeccable but that his prose is terse and unambiguous and his symbolism well-planned and such as to strike the eye effectively. Those authors are at an advantage who can visualize how their formulae are likely to appear on the printed page.

One concluding excerpt concerning possible future trends may also be of interest:

At this time there is strong advocacy to displace the printing of mathematics by photographic methods applied to typescript or manuscript. There is no doubt a place for them in (say) the duplication of lecture notes or in reproducing a doctorate thesis *in extenso*. But it is difficult to believe that these processes can ever satisfy the present demands of mathematicians for the rich arabesques of symbolism they are driven to employ. If increasing complexity of mathematical notation sends up costs so much that publishers are driven more and more to photographic methods, the prospect for mathematicians is bleak. To help postpone this day is one of the purposes of this book.

Mathematics for pleasure with profit . . .

My readers need hardly be reminded of the universal and ageless interest manifested in mathematical recreations. Scarcely a few months go by without the appearance of another book in this area. Yet styles change, even here. Within a hundred yards or so from my study there stands a historic house, nearly three hundred years old. What kind of books were written in those far-off days, when the art of printing from movable type was still a comparative novelty? Before me lies a book printed in England some thirty years later—in 1694, to be exact—and written by one William Leybourn.

Unlike modern books on mathematical pastimes and puzzles, the tracts of those days devoted a great deal of space to so-called recreations in the physical sciences —notably chemistry, natural philosophy, mechanics, astronomy and horology. Our friend William Leybourn was no exception. To convey the flavor and nostalgia of a work two or three centuries distant in a few brief pages seems well nigh impossible; but I shall attempt it, nevertheless, by giving two excerpts: (1) some of his observations on the nature of recreations, and (2) a piece taken from his "Statical" recreations.

To the **INGENIOUS READERS:**
and principally
To such as are
MATHEMATICALLY Inclined.

RECREATION, (saith a late Learned Divine) is a second **CREATION,** when Weariness hath almost annihilated our Spirits: It is the *breathing* of the *Soul,* which would otherwise be *stifled* with continual *Business* . . .

He then continues:

As *Recreations* and *Exercises* (whether of *Body* or *Mind*) are various: So one *Dish* of *Meat* may as well please all *Palates;* as one *Recreation* suit all *Dispositions:* And (as I have said elsewhere upon the like occasion), That, although some *Corporeal Exercises* may to some *Bodies* tend more to *Health,* and some *Mental Labours* more to *Wealth;* yet none affords the *Mind* more *pleasure* with less repentance, than those of the Mathematicks do.

Our mentor then discourses briefly upon "a few corporeal exercises," including bell-ringing; fishing with the angle; hunting; fowling; running, leaping, dancing and walking (excellent when used in the morning); swimming; and bowling. In connection with bowling, he remarks:

It teaches Mens *Hands* and *Eyes Mathematical Proportion*:—And (for a Home-Diversion) the Play at the *Billiard Table* hath not its Peer: It exercises the whole *Body moderately;* the *strength* of the *Arm judiciously:* It *directs* the *Hand Geometrically,* and the *Eye Optically:* For the attaining to be an Exquisite Proficient in *playing* at it, depends wholly upon putting in *practice* that *Axiome* of *Euclid* in his Catoptignes; which *demonstrates,* that, *The Angles of Incidence and Reflection are ever more equal.*

Pleaſure with Profit:

Conſiſting of

RECREATIONS

OF

DIVERS KINDS,

VIZ.

Numerical,	*Aſtronomical,*	*Automatical,*
Geometrical,	*Horometrical,*	*Chymical,*
Mechanical,	*Cryptographical,*	and
Statical,	*Magnetical,*	*Hiſtorical.*

Publiſhed to Recreate Ingenious Spirits; and to induce them to make farther ſcrutiny into theſe (and the like) SUBLIME SCIENCES.

AND

To divert them from following ſuch Vices, to which Youth (in this Age) are ſo much Inclin'd.

By WILLIAM LEYBOURN, Philomathes.

To this Work is alſo Annext,

A TREATISE of ALGEBRA,

According to the late Improvements, applied to *Numerical Queſtions* and *Geometry*; with a NEW SERIES for the ſpeedy *Extraction of Roots*; as alſo a CONVERGING SERIES for all manner of *Adfected Equations.*

By R. SAULT, Maſter of the *Mathematick School* in *Adam's* Court, in *Broadſtreet*, near the *Royal Exchange*, *LONDON.*

LONDON:

Printed for **Richard Baldwin**, and **John Dunton**; near the *Oxford-Arms* in *Warwick-Lane*: And at the *Raven* in the *Poultrey*. 1694.

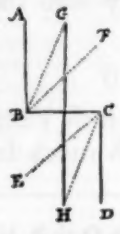
R. Chubb.

With an even briefer nod to the more "heroical" corporeal exercises fit only for the recreations of princes, such as horsemanship, tilting, tournamenting, throwing the bar, wrestling, etc., he proceeds thus:

. . . But leaving those of the *Body*, I shall proceed to such *Recreations* as adorn the *Mind;* of which those of the *Mathematicks* are inferior to none. Now the *Excellency* of any *Science* (says the Philosopher) may be judged of, (1.) by the *Excellency* of the *Object:* And (2.) by the *Certainty* of its *Demonstrations. First,* for the *Object;* It is no less than the *whole World:* Not only of the Terrestrial, but the Coelestial part thereof also. . . . *Secondly,* For the *Demonstrations* of these Sciences, they are as infallible as *Truth* it self: And for this reason also doth it exceed all other *Knowledge* which depend upon *Conjectures* and *Uncertainty.* . . . And thus let what I have already said concerning the *Excellency, Utility* and *Benefit* of these *Mathematical Arts* suffice.

* * * * *

How is it, that a Man sitting in a Chair, his Back to the back of the Chair, and his Legs by the fore frame of the Chair upon the Ground, cannot rife off the Chair except he bend his Body forward, or put his Legs backwards.



IN the Poſture of Sitting, our Legs are ſuppoſed to make a Right Angle with our Thighs, and they with our Backs: As in the Figure.

Wherein, Let A B repreſent the Back; B C the Thighs; and C D the Legs. It is now evident, That a Man cannot riſe from this Poſture, unleſs, either the Back A B, do firſt incline unto F, to make an acute Angle with the Thighs B C; or elſe that the Legs C D, do encline towards E, which may alſo make an acute Angle with the Thighs B C: Or, laſtly, unleſs both of them do decline to the Points G and H, where they may be included in the ſame Perpendicular.

For the reſolution of which, Philoſophers give theſe two Reaſons:

1. A Right Angle (ſay they) is a kind of Equality, and that being Naturally the cauſe of Reſt, muſt needs be an impediment to the motion of Riſing.

2. Becauſe, when either of the Parts are brought into an acute Angle, the Head being removed over the Feet, or they under the Head, in ſuch a Poſture the whole Man is much nearer diſpoſed to the form of ſtanding, wherein all theſe Parts are in one ſtreight Perpendicular Line, then he is by the other of Right Angles, in which Back and Legs are two Parallels; or that of turning theſe Right Angles into Obtuſe, which would not make an erect Poſture, but declining rather.

Theſe are the Philoſophical Reaſons; the Geometrical may be theſe. Suppoſe B C to be a Leaver, towards the middle of which is the place of the Fulciment, Prop or Center, A B as the Weight, and C D, the Power which is to raiſe it.

Now the Body being ſituate in this Rectangular Form, the Weight A B muſt needs be augmented proportionably to its diſtance from the Center, which is about the middle of the Thighs; whereas, if we ſuppoſe either the Weight to be inclined unto F, or the Power C D to E, or both of them to G and H, then there is nothing to be lifted up but the bare Weight it ſelf, which in this ſituation is not at all increaſed with any Addition by diſtance

Albert Einstein: man and legend

It is doubtful whether the work of any other single man has made so profound an impact on the life of the twentieth century as that of Albert Einstein. He has become a legend during his own lifetime. The literature concerning the man and his work is already extensive, two very readable and worthwhile additions having appeared late last year.[2]

Ideas and Opinions represents an attempt to gather together, between the covers of one book, the most significant of Albert Einstein's general writings. Up to this time there have been published three major collections of articles, speeches, statements and letters by Einstein: *The World As I See It,* translated by Alan Harris (1934); *Out of My Later Years* (1950), which contains material from 1934 to 1950; and *Mein Weltbild,* edited by Carl Seelig, published in Switzerland (1953), containing some new material not included in either of the other two volumes. According to the publisher, *Ideas and Opinions* contains the most important items from these three books, together with a few selections from other publications and some new articles that have never before been published in book form.

A rather extensive collection of more than 120 items, it is divided into five parts. The first section includes ideas and opinions about life in general, about freedom, about religion, about education, and about some of his friends. Section II deals with politics, government, war and peace, and pacificism; Section III, some observations on the Jewish people; Section IV, five brief pieces on Germany. Section V contains nearly two dozen excerpts dealing with science. In the opinion of this department, eminently worth reading are: "What Is the Theory of Relativity?"; "Geometry and Experience"; "Maxwell's Influence on the Evolution of the Idea of Physical Reality"; "On the Method of Theoretical Physics"; "Physics and Real-

[2] *Ideas and Opinions,* Albert Einstein, translated by Sonja Bargmann (New York: Crown Publishers, 1954), 377 pp.

The Drama of Albert Einstein, Antonina Vallentin, translated by Moura Budberg (Garden City: Doubleday & Company, 1954), 312 pp.

ity"; "The Fundamentals of Theoretical Physics."

The Drama of Albert Einstein is a modest but distinguished biography—an intimate portrait of a truly great man. It is a wonderfully warm and human story, written with keen insight and deep affection. Not only is the personal side of a dramatic career graciously unfolded; the inexorable events that ushered in the atomic age are also told in crisp moving words. Thus:

On August 12, 1939, Albert Einstein wrote a letter. Its address: The President, White House, Washington. Its contents were simple:

"The results of the research recently pursued by E. Fermi and L. Szilard, submitted to me in manuscript, have revealed that we may in the immediate future expect to find the element uranium capable of being transformed into a new and considerable source of energy. This new phenomenon may also lead to the construction of excessively powerful bombs. A single bomb of this type, transported by ship and allowed to explode in a port, could destroy the whole port and the surrounding territory."

A. Einstein

The atomic age was born.

We were particularly impressed with the genuine understanding shown of the spiritual side of the man. For example:

Einstein's principal source of energy, which allows him to defy the deterioration of his body, is the feeling of this mission that he has still to accomplish. . . .

The universe of Albert Einstein is governed in the twilight of his life by this moral obligation. He submits to a force which he sometimes calls "reason displayed in life," which in its more intimate depths is inaccessible to man. More and more often he calls it God. Not the personal and revengeful god of his ancestors, but the God of the supreme order of nature, who leaves nothing to chance. It is God who has given him the faith that made him persevere in his research, alone, and attacked even by those who were closest to him in thought.

Posterity will judge whether the years of incessant effort, of defeats overcome, of patient and obstinate reconstruction, were in vain; it will judge whether the theory of the unified field represents the triumph of human spirit over the chaos of the universe. This triumph would be a triumph of faith, a faith which across spiritual victories and defeats will remain with him while he is alive. One day in a conversation Einstein summed it up in a sentence that made a great impression on his questioners, for it seemed to be a beacon capable of guiding searchers through the anguish of their exploration of the universe, through their battle for truth. This sentence expresses also the meaning of Einstein's struggle against man's folly and for the survival of humanity. It is now engraved over the mantlepiece of a room in Fine Hall at Princeton: "God is subtle but he is not malicious."

Words! Words! Words!

A variable! What is it? Is it something that changes and slithers around under the cover of an X or under the cover of a mental haze brought about by too much emphasis on the mechanics of algebra?

Often secondary pupils are told that a variable is something that changes; like temperature and the weather. They are led to recall experiences with temperature (or something similar) and then told that temperature is a variable, and the letter d in the formula $c = \pi d$ is also a variable, since it may represent any number. Nothing could give the pupil a more erroneous idea of the way the term *variable* is used in mathematics. The temperature experience is not a suitable experience in this instance since it does not fit the way mathematics uses the term *variable*.

A variable is essentially an arbitrary member of a set of numbers, and this arbitrary member is symbolized by a letter of the alphabet, usually x, y or z. Hence, if the pupil is to get the idea of what the teacher has in mind when the word *variable* is used, it is necessary to introduce the concept of a set of numbers. For example, if one were to write down the set of numbers one of which might represent the attendance for the day in a specific algebra class on March 30, 1955, the set would consist of the numbers 0, 1, 2, 3, 4, · · · , 30, 31, 32, 33, 34, where 34 is the number of pupils enrolled in the class. Now, if some computation is to be made involving the number of minutes spent working on algebra on March 30, it is desirable to agree that X is to represent any one of the above set of numbers and proceed with whatever algebraic expression the problem situation requires. Note that, as of March 15, 1955, the letter X is a symbol which represents an arbitrary member of a specific set of numbers; namely the actual attendance on March 30. The letter X does not slither around; it is a symbol representing ANY member of the set.

If we are to teach algebra meaningfully, the time must come when teachers will fit the experience to the idea; as yet this has not happened in too many classrooms. In this instance it cannot happen unless the concept of a set of numbers is given considerable attention.

—Henry Van Engen

Reviews and evaluations

Edited by Richard D. Crumley, University of South Carolina, Columbia, South Carolina, and Roderick C. McLennan, Arlington Heights High School, Arlington Heights, Illinois

BOOKS

The Slide Rule, J. N. Arnold, Prentice-Hall, Inc., New York, 1954. Cloth, 206 pp., Library of Congress No. 54-11640.

This book gives a very thorough treatment of the slide rule. The first 85 pages may be used with ninth-grade pupils. There is very little material that cannot be mastered by superior high school pupils. The text will also be of great value to pupils in engineering schools, and for people who wish to learn how to use the slide rule without the help of a teacher.

The text gives a good treatment of the "correction number" method for locating the decimal point. This is by far the best method for treating this all-important phase of slide rule computation.

The last 50 pages are devoted to applications of the slide rule to business, finance, statistics, chemistry, physics, and engineering.

It is unfortunate that only two pages are devoted to computation with approximate data. The slide rule is the most important instrument for computation with approximate data and is best understood when this most important topic has been thoroughly mastered.—*Carl N. Shuster, Trenton, New Jersey*

Algebra Book 2, C. A. Smith, W. Fred Totten, and Harl R. Douglass, Evanston, Illinois, Row, Peterson and Company, 1954. Cloth, vi+502 pp., $2.72.

It is certainly difficult to judge a book for someone else's use. Type and arrangement of exercises, number of pictures, order of topics, and similar considerations may depend upon the personal teaching habits of the user; but there are still certain principles which can be sought and agreed upon in any book. The book now being reviewed does not *completely* satisfy any one of the following principles, although there are many good features. A textbook

(1) should contain no mathematical mistakes.

(2) should use helpful and clear definitions among all the correct ones available.

(3) should appeal to pupils; that is, should motivate learning.

(4) should tell WHY mathematics works the way it does, as well as HOW it works.

(5) should have concern for logical organization and structure of the presentation.

(6) should apply the mathematical procedures to practical and real situations. (This does not mean that some artificial applications may not be allowed, if they are recognized and pointed out.)

These six statements must not be taken as a definition of all that a good textbook needs to be. They are merely the headings under which criticisms of the present book are organized.

Mathematics: The mathematical mistakes here discussed will be collected under six headings: numbers, signs, radicals, factors, equations, and graphs.

There is a constant confusion between the meanings of quantity and number (pp. 42, 52, 54, 120, 121).[1] In treating significant figures the expression "less than one half unit" (p. 213) is incorrect since 12.4 is correct to the nearest .05 of a unit. The rules for adding approximate figures will lead to wrong answers (pp. 215, 216) except in the specially cooked-up examples used in the book. Logarithms (p. 265) are not clarified by saying that they are computed "by methods such as those illustrated"—they are not!—nor is a serious student instructed by telling him that a "small percentage" of all numbers can be written as an integral power of 10. Finally, although most encounters with infinity are well handled, once (p. 288) we are told that the "value of x approaches infinity."

In general the signs + and − are well explained, but there is some confusion (p. 5) between the use of + and − as operations, as indications of direction, and as inverses. In synthetic division (p. 33) the rule should not say "a with its sign changed."

Radicals are known to be full of traps for both student and teacher. We are told (p. 35) that "any number has two square roots" at a time when this statement is false; negative numbers have been introduced, but not imaginary numbers. The rule $\sqrt[n]{x^n}=x$ (p. 227) is not true if n is even and $x<0$, and, although correct at the place where it is given, is not corrected later when imaginary numbers are introduced. The same can be said for the rule $\sqrt[n]{x}\cdot\sqrt[n]{y}=\sqrt[n]{xy}$ (p. 228). Another statement is made (p. 35) which is wrong as it is stated: $\sqrt{16x^2}=4x$. What about the value of $x=-3$, which is not excluded? The justification (p. 240) for being able to say $5\sqrt{6}=\sqrt{150}$ is completely fallacious since

[1] Numbers in parentheses will mean pages in the textbook which illustrate the remark.

the second expression really does not have any more significant figures than the first, although it may superficially seem to have.

The definition of *factor* (p. 7) is insufficient to cover example 11 on page 13, since *prime factor* is not yet introduced. When the concept of *prime number* is introduced (p. 80) 5 is not prime by either the definition on page 71 or the definition on page 7, since $5=(2)(\frac{5}{2})$ or $5=\sqrt{5}\sqrt{5}$. To add to the confusion, we *must* be able to call these expressions "factors" if we are permitted the definition of *square root* used on page 35. In giving rules (pp. 69, 75) for $(a+b)^2$ and $(a-b)^2$ we find actual contradictions; it is not clear whether the second term of $(x-y)$ is y or $-y$. The distributive law (p. 8) is so obviously connected with monomial factors that it is disappointing (p. 17) to see a false explanation of this type of factoring using the "apple theory."

The subject of *equations* falls down in a few places. It looks as though (p. 133) "linear" is considered equivalent to "one variable increases as the other variable increases." In the discussion of variation (p. 141), "d varies directly as r" is equated to "when r increases, d increases, and when r decreases, d decreases" which is correct; but then it goes on to equate these to "d is directly proportional to r" which is wrong. In another place (p. 142) the expressions "varies inversely" and "is inversely proportional to" are considered synonymous. It is often improper to carp at the use of the word "transpose," but the statement "we transpose -28" in the equation $x^2-3x-28=0$ (p. 348) causes confusion. Some who use the word will feel we "transpose" 28. Far along in the book (p. 379) we are told "we begin study of quadratic equations in two unknowns"; yet the graphs of $y=ax^2+bx+c$ have already been extensively studied.

Turning our attention to *graphs* we might want to discuss rather than proclaim (p. 133) that "the graph is the most complete way of expressing the functional relationship"; since few graphs can be shown completely on a finite piece of paper, why is the equation not "the most complete" way, if that expression is necessary? It is quite incorrect to say (p. 160) that the procedure for justifying equations of lines parallel to the y-axis is similar to that for those parallel to the x-axis, especially when the latter have been explained in terms of slope. One of the craziest mistakes is the explanation (p. 274) that linear interpolation is possible with logarithms because "as the graph is extended it becomes almost a straight line." Is it not misleading (p. 334) to say that a parabola is the graph of an equation in one unknown?

Of course it is difficult to state mathematical facts with complete accuracy in a short space, but it is not fair to introduce mathematical mistakes, for the intuition of the learner will be confused even if he accepts the authority of the printed statement.

Definitions: Sometimes definitions, while not really incorrect, are not as helpful as they could be. Some definitions in this book will be criticized under these four categories: (1) The word is not defined or explained before it is used, or the meaning is changed without warning, (2) An explanation seems to suggest a definition which is misleading, (3) The choice of definition shows poor judgment, or (4) Words are defined which had better be omitted since they are now old-fashioned, not used, and therefore probably defined through habit rather than need.

The word "term" is used (p. 11), but it is neither described nor defined. Negative definitions such as "An expression which is not integral is fractional" (p. 140) are valid only if the larger class, expression, has already been defined. We have already commented on the confusion over the meaning of "factor," but its meaning is changed (p. 251) and the new one never clearly stated. For example, why should we say that $a+b=(\sqrt{a}+\sqrt{b})\ (\sqrt{a}-\sqrt{b})$, why not $a+b=(\sqrt{a}+\sqrt{b})\ (\sqrt[4]{a}+\sqrt[4]{b})\ (\sqrt[4]{a}-\sqrt[4]{b})$ or even more factors? Finally, the whole subject of "circular permutations" is poorly introduced (p. 447); the words are never explained or defined, and the text and diagrams leave things completely vague.

In introducing rectangular coordinates (p. 149) the words "right angle" are not used. The word "indicates" is a weasel word which is indefinite enough not to be incorrect, but its very indefiniteness leaves it open to misinterpretation; for example, "A fraction indicates the division of one number by another" (p. 94) and "The degree of an equation indicates the number of its roots" (p. 88). Are these definitions, theorems, coincidences or what? A similar confusion is found with "Ratio is a comparison by division" (p. 120) and it is even set off as though this were a definition for "ratio." Dictionaries were not intended to have only mathematical application and their statements are seldom helpful in mathematics classes; for example, in discussing formulas what good does it do to say that "The dictionary gives 'action' as one meaning of function. How is the value of d an 'action' of r" in $d=5r$ (p. 139)?

Sometimes the definitions merely show poor judgment. Equations should not be called "simultaneous" only when they have a root (p. 169); why not call such equations consistent, and let "simultaneous" cover any equations which are considered simultaneously, even if they are inconsistent or dependent. The distinctions attempted between dependent and independent variables (pp. 52, 131) are not at all helpful to a beginner; the real distinction is not one of necessity but one of choice. Further, it is misleading to say (p. 131) that in the formula $d=rt$ there is no constant; how do you know? Later (p. 147) the same formula is used with r a constant! Also, it is unfortunate to define polynomial (p. 12) as having more than one term; a monomial is a special case of a polynomial just as a binomial, etc.

Some words and symbols should have been discarded as no longer needed in algebra. Here are some examples: vinculum (p. 23); simple

equation (p. 43); LCD in addition to LCM (pp. 82, 102); 2:3::6:9, extremes, means, terms (of a proportion), fourth proportional (p. 122); abscissa, ordinate (p. 149); surd (p. 226); and, above all, conjugate binomial quadratic surd (p. 237).

Motivation: The procedure of the textbook seems to be completely mechanical, and the attitude excludes inductive approaches; first give the rule, and then apply it. Teachers may not teach that way, even with this book, but judging the book itself (since some teachers do follow a book closely), the spirit of the book must be attacked in this review.

The solving of verbal problems is certainly mechanical (pp. 54, 59) since rules and "table methods" are indicated. The solution of simultaneous equations (p. 171) gives no indication of WHY the process developed, but just gives rules. The same is true of the introduction to scientific notation (p. 217) and to the process of finding square root (p. 224). The "foil" method of multiplying binomials (pp. 68, 236) shows a concern for mnemonics rather than thought.

An inductive approach to "functional changes" rather than an unmotivated "principle" (p. 135), would be better. The introduction to imaginary numbers (p. 247) would help no student who did not know the subject before he approached the book; and the graphing of imaginary numbers indicates no connection with the graphing of real numbers on a straight line (p. 253); such motivation would avoid the pupil's reaction, "But who wants to graph imaginary numbers?"

Logarithms are difficult to motivate before they are defined, but they do not need to be presented as a completely mechanical process (p. 262). For example, the subject of negative characteristics (p. 271) is justified by the direction in which you move a pencil in counting, but never by referring to the definition of logarithms.

Why: Why certain definitions are given, why certain rules and procedures are used, why certain topics are included in the course, are just as important to the student as how to define, how to proceed, and how to remember what comes next in the book. This is closely connected with the topic of motivation, but all motivation is not identical with telling "why"; on the other hand, sometimes we tell "why" to maintain the logical pattern, not merely to motivate.

Here are some concepts which are introduced without telling why: LCD (p. 48), the distinction between $tu=(t)\times(u)$, but $35\neq 15$ (p. 179), and several words connected with the parabola (turning point, minimum, maximum, etc.) (pp. 334, 335).

Many, many procedures are given with no proper explanation of "why" each is a good or acceptable procedure: the explanation that a minus times a minus gives a plus (p. 8); coefficients in front of parentheses are always 1 (p. 22); or even why every parenthesis need have a coefficient (p. 22); the procedure of arranging polynomials in descending order before multiplying (p. 25); the rule "invert denominator and multiply" (p. 100); the reason why $m>n$ in the rule for exponents (p. 208); the reversal of factors in the rule $(x^a)^m=x^{ma}$ (p. 209); the procedure for deciding how many significant figures to keep in an answer (p. 215); any reason why one should want to simplify radicals even if he knows how (pp. 228–234); the procedure for adding and subtracting radicals (p. 234); the definition of functions of obtuse angles (p. 322); and others. It is greatly to be doubted whether rules which are not capable of justification are worth teaching. As an example of a place where the "why" *is* given we might cite the rule for marking off pairs of digits before using the method of finding square root (p. 223).

Seldom is continuity given to the development of the mathematics by showing why a new subject is introduced. For example, second and third order determinants are defined and evaluated, and then it is shown that they help in solving simultaneous equations (p. 195, 198).

Logic: In an elementary book the logical steps in proceeding from one part of mathematics to another must often be sketchy, but they should never be incorrect or seem to indicate a logical direction which is fallacious. Such shortcomings will be discussed here with regard to four topics: powers and roots, solving equations, checking equations, and extraneous roots.

The logic of $(x^m)^n=x^{n^m}$ is confused (p. 34). Although the rule $(\sqrt[n]{x})^m=\sqrt[n]{x^m}$ is correct when first given (p. 221), it is not changed later (pp. 247–256) when it has become incorrect. The whole approach to the extensions of the meaning of "exponent" is filled with danger; it is far from clear that we are defining new types of exponents, rather than proving that they *must* have the meanings assigned. This is true for zero (p. 211), negative (p. 211) and fractional (p. 221) exponents.

In solving equations the teacher must decide whether transposition is to be encouraged, allowed, or condemned; this book does not help much with that decision (p. 46). Here is another minor point, but it is to be wondered what discussion will result from this sequence (p. 88):

(1) $(3x+1)(x-2)=0$
(2) Either $(3x+1)=0$ or $(x-2)=0$
(3) Both $-\frac{1}{3}$ and 2 are roots.

The steps given for solving equations (p. 88) are not complete without another step: Check to see if they *are* roots.

The whole subject of checking roots is poorly handled (pp. 43, 45, 103). Starting with an equation to be proved and then operating on it as though it were true, is poor logic. Whether the words are used or not, the concept of necessary and sufficient conditions would help the student out of the logical troubles (p. 169) which confuse the approach to simultaneous equations.

The use of necessary and sufficient conditions would also remove the illogical approach to "extraneous roots" (p. 110) which is better explained by reversible and non-reversible operations, than by the discussion which merely makes

you feel that sometimes algebra lets you down (p. 242). The phrase "introducing and losing roots" (p. 355) also makes algebra sound very unreliable, which it is not.

Applications: Perhaps an honest distinction between artificial applications that are both fun and instructive and practical applications that show how people really use mathematics would avoid the silly problems which masquerade as practical applications.

Instead of analyzing all the problem material, which would take too much detail, it might serve to inspect the pictures (omitting the graphs and purely geometric illustrations). They can be divided into diagrams and photographs. Submitting them to several teachers for evaluation gave the following figures:

	Excellent	Good	Fair	Poor
Diagrams	11	35	33	8
Photos	1	2	5	7

For example, some pictures were completely dishonest and unconvincing applications of mathematics, pages 65, 66, 128, 129, 205, 301, 440, and 441. The drawing at the top of page 379 showing conic sections and their uses could much better have been expanded to cover two or three pages, and have included some fine photographs, and have replaced some of the useless material. Sometimes the pictures are not poor applications, merely confusing (pp. 15, 206).

Good points: It must be indicated that some features of this book are good. It is easier to find fault, and more revealing, than to tell all the material which is well done. For a few examples: the order of topics is very teachable; the diagram of *base* and *exponent* (p. 10) is good teaching; many of the clever diagrams beside the word problems are entertaining and instructive (for example, pp. 115, 116); the introduction to analytic geometry is quite satisfactory (pp. 150–163); a careful explanation of the meaning of a radical sign, and the distinction between $\sqrt{9}$ and $-\sqrt{9}$ is given (p. 220); the parallel column arrangement of numerical and general cases for the solution of the equation $ax^2+bx+c=0$ (p. 352) is appreciated; and the discussion of graphical solution of quadratics (pp. 391–397) is good because it takes time to be leisurely and illustrate its points. Balancing all opinions, however, this is not a top-notch book for teaching second-year algebra.—*Henry W. Syer, School of Education, Boston University, Boston, Massachusetts*

Trigonometry, Elbridge P. Vance, Addison-Wesley Publishing Company, Inc., Cambridge 42, Massachusetts, 1954. Cloth, viii+158 pp., $3.00.

As in many recent texts, the emphasis in this book is on analytic rather than numerical trigonometry. The general circular functions are defined in terms of a coordinate system and a substantial portion of the theory is treated before the solution of triangles. In the last chapter the student is introduced to some modern applications of trigonometry through brief discussions, on an elementary level, of simple harmonic motion and harmonic analysis with illustrations from electricity and sound.

The text is concise and well written. In the reviewer's opinion, it has several good features. One section on graphing of the circular functions appears immediately after their definitions. The material on logarithms is relegated to the appendix, which enables the instructor to omit it or not depending on the purposes of the course and the background of the class.

Four-place tables of logarithms, trigonometric functions and logarithms of the trigonometric functions are included in the appendix. —*Fred W. Lott, Iowa State Teachers College, Cedar Falls, Iowa*

Elementary Statistics, John M. Howell and Ben K. Gold, Wm. C. Brown Company, Dubuque, Iowa, 1954. Paper, v+154 pp., $3.00.

You will be favorably impressed when you first leaf through this elementary statistics book. As you read the text, you find it is well written. The statistical concepts are thoroughly explained. There are many practical problems, but very few answers are given for the problems.—*O. C. Kreider, Iowa State College, Ames, Iowa*

College Algebra, H. G. Apostle, Henry Holt and Company, New York, 1954. Cloth, v+432 pp., $4.50.

This text is an excellent example of a growing trend to base the study of algebra on its basic axioms and postulates, and then to develop the subject matter scientifically by logical processes, definitions, theorems, etc. The author has tried to present the conventional topics of algebra as logical principles of science, employing both inductive and deductive methods.

In the early chapters the student gets a generalized concept of college algebra as a science. The basic assumptions and definitions are introduced and carefully presented—12 axioms for natural numbers, 26 axioms and 14 theorems for real numbers are considered. In the later material, whenever a rule is stated the author points out from which axiom and/or theorem it is taken.

A rather complete treatment of the Cartesian coordinate system and the techniques of graphing should prepare the student for later work in analytic geometry. Stress placed on maximum or minimum values of functions of the form $y=ax^2+bx+c$ should make differential calculus a little less foreboding. In the many illustrative examples, the solutions include a good discussion of the elements of the problem involved before the mechanics of the solution appear. The problems are fine, with numerous applications from the fields of physics, mechanics, engineering, commerce, etc. A very good chapter on "Theory of Equations" includes general solutions for the

cubic and quartic, and a detailed discussion of Horner's Method.

The text is a lengthy one but primarily because of the completeness with which the subject matter is treated. This is one of the few recent books which can really be thought of as a *new book* in college algebra. It merits most careful consideration as a text for classroom use.—*Irwin K. Feinstein, Chicago Undergraduate Division, University of Illinois*

DEVICES

Perimeter Area Board (#763), Ideal School Supply Company, 8326 Birkhoff Avenue, Chicago 20, Illinois. Cork-faced board, 10″×20″, with 20 large pins, 6 rubber bands, and one 20″ measuring tape; 8-page instruction manual included; $2.50.

The cork face of the board has a checkerboard design with 1″ squares. The pins can be stuck in the board anywhere and enclosed with a rubber band to form a geometric figure. The squares on the board enable the student to find the area, and the measuring tape can be used to find the perimeter.

This device is well constructed and can be used effectively in grades five through eight. By means of this board and accessories, students can discover for themselves how to find the area and perimeter of rectangles, squares, and right-angle triangles. The board can also be used to learn how to multiply and divide with whole numbers and fractions. The instruction manual written by Elda L. Merton (who also designed the board) provides some suggestions for the use of the board.—*Richard D. Crumley*

Spinner Fraction Pie Game (N248), Creative Playthings, Inc., 5 University Place, New York 3, New York, or 316 N. Michigan Avenue, Chicago 1, Illinois. Game composed of wooden spinner, 6 rubber pies cut into fractional parts; $2.95.

This device consists of six rubber pies ($\frac{3}{4}$″ thick) cut into thirds, sixths, twelfths, halves, fourths, and eighths in different colors; an 11″×12″ framed wood board with sliding cover and six 3″ circular holes for the rubber pies; and a hexagonal wooden spinner with the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$ printed on the six sectors.

A nice feature of this device is the flexibility with which it can be used. The rubber pies and the board with holes for the pies can be used by small groups of students to discover and understand the relationships between fractions without using the spinner to make it a game. A two-page, mimeographed set of instructions accompanies this device and describes a couple of games that may be played, and other games could be invented using the device. Even though such instructional material would most commonly be used in the fifth and sixth grades, it could be used effectively for remedial instruction in later grades.—*Richard D. Crumley*

FILM

Weights and Measures, Encyclopaedia Britannica Films, Inc., 1150 Wilmette Avenue, Wilmette, Illinois. 16 mm. film; black and white; 14 min.; $62.50.

Designed for the middle grades and junior high school arithmetic and general science classes, this film deals with the importance of weights and measures by showing some experiences of a young junior high school student. Linear measure, area measure, and volume measure are illustrated, as well as different measures of weight. The collaborator for the film was John R. Clark, and the producer was Milan Herzog.

The photography and sound of the film are excellent. Animation is well integrated with the live photography and is effective. Things done in the film could not be done easily by a teacher in the classroom, so the medium of film is used to good advantage. This film will serve well as an introduction to an instructional unit on weights and measures; however, it can be used for other purposes, too. A nice feature is the emphasis given to the suitability and selection of different units and measuring devices according to the purpose of the measurement.—*Richard D. Crumley*

Have you read?

LAMERS, WILLIAM. "A Friendly Classroom Atmosphere." *The Clearing House*, Vol. 29, September 1954, pp. 46–47.

All of us hope for a friendly classroom and realize that learning increases in a geometric ratio under such pleasant conditions. But how to achieve this is another problem. This short article gives some potent things to think about; for example, closing the door to yesterday; never nurse yesterday's wounds; smile; please don't scream; keep looking up; put sugar on sour apples, and several others. This is well worth reading, especially on one of those blue days. —PHILIP PEAK, *Indiana University, Bloomington, Indiana*

Edited by John A. Brown, University of Wisconsin, Madison 6, Wisconsin, and Houston T. Karnes, Louisiana State University, Baton Rouge 3, Louisiana

Subtracting signed numbers

by Alice Entwhistle, Canton High School, Canton, Illinois

I shall attempt to explain in this article some of the illustrations, examples, and procedures I use in presenting the subtraction of signed numbers to my classes.

In introducing signed numbers I stress the fact that the minus sign is a symbol. This symbol has several meanings. It means subtract as in arithmetic, and also is the sign of a directed number. When it is the sign of a number, as in -3, then the symbol indicates that the number -3 is the opposite of the number $+3$. In problem situations the negative number may mean: (a) below the starting point (0); (b) to go in the opposite direction; (c) less than zero (below). Zero is always the point of comparison from which to start, when plus and minus signs are used.

Some examples that illustrate direction are listed here. I have two students start from the center of the room. One student will walk west five steps. He will tell the class his final position from the center of the room by writing on the board the symbol "-5." The second student will walk five steps east and will tell the class his final position by writing the symbol "$+5$."

I sometimes drop a ball from the desk and toss a ball up from the desk. The students then describe the direction of the ball by the plus and minus symbols.

Money that is saved and owed also furnishes good examples because most students have had direct experiences with these situations. Games where scores can be less than zero are effective to use as illustrations and drill exercises for adding and subtracting signed numbers.

I begin the instruction of subtraction of positive and negative numbers by using money. I hold a \$10.00 bill in my hand and make a \$15.00 bet with a student; and I lose. I pay the \$10.00; but I still owe \$5.00. This situation is then described by

$$+10-(+15)=-5.$$

The first minus sign means subtract. The second minus sign is a part of the number -5 and means here that I owe \$5.00.

I also use this example. Bill's bank balance is \$2.00 overdrawn at the bank. Bill writes a check for \$10.00 the next day so his account is then \$12.00 overdrawn. In this situation the words "write a check" mean subtract. The words "\$2.00 overdrawn" and "\$12.00 overdrawn" are represented by the negative numbers -2 and -12. This situation is then described by

$$-2-(+10)=-12.$$

I illustrate the subtraction of two negative numbers by using this example. Jane owes Mary \$12.00. She then pays Mary \$10.00. That is, Jane has a debt of \$12.00 and she then subtracts a debt of \$10.00. She now has a debt of only \$2.00. Thus

$$-12-(-10)=-2.$$

In selecting examples, I am sure to use words that mean *subtract* and *in the opposite direction.*

From the examples, I let the pupils formulate their own rules, and most of them soon discover for themselves the shorter rules for subtraction. The rules have meanings for them and are not merely memorized statements that they repeat.

I teach all the rules for the four operations with signed numbers without the use of a textbook. I give my students mimeographed problems for their assignments. Problems in the textbook are not assigned until the students have mastered the rules. Since I have been using this method of presentation, I find that the number of difficulties encountered by the students have been very few. I believe this due to the fact that the student has worked out his own rules and relies upon an understanding of the principles rather than memorized rules written by someone else.

I also use cards on which statements such as the following are written:

"Subtract 10 from your score."

"Take −3 from the person on the right and subtract it from your score."

"Subtract from your score the number that will make the score of the person on your left equal to zero."

I have about fifty cards in a set and enough sets for each class. About five students play in a group. Scores are computed at the end of a certain time, or at the end of a certain number of draws. The students have fun learning their addition and subtraction facts in this manner.

Suggestions for counseling in high-school mathematics

by William L. Hart, University of Minnesota, Minneapolis, Minnesota

On many occasions, I have expressed the opinion that counseling in secondary mathematics should be based on the mathematical ability of the student rather than on his intentions of the moment with respect to a future vocation, or future collegiate training. That is, I believe that the student of high mathematical ability will gain more good, aesthetic and practical, from the continued study of substantial mathematics than he would from any mathematical course of a weak variety, or any other substitute course of a non-mathematical type, regardless of his future vocation or profession. The student of ability who studies a great amount of substantial mathematics in high school, but never goes to college, more frequently appreciates his mathematical training, and sometimes obtains more tangible rewards as a result of it, than some students of similar ability who go on to college. I wish it to be understood that my following remarks relating to college contacts do not contradict the preceding creed that the substantial parts of the curriculum in secondary mathematics should be planned for the student of requisite ability, whether or not he intends to enter some college.

When the counselor in a high school debates advising some student to take one or more of the advanced courses in mathematics, higher algebra, trigonometry, and solid geometry, naturally the counselor considers not only the mathematical ability of the student but also all other pertinent features. Among these we may mention the psychological reaction of the student to the subject—does he like it or does he dislike it, even though he is successful in its study; does the student aim at some trade, or some collegiate curricu-

lum where advanced secondary mathematics will be useful? It is my purpose to emphasize certain facts about curricula at the University of Minnesota which the counselor should keep in focus when advising the student of requisite ability.

It is common knowledge that any student who intends to enter a technical college should study as much mathematics as possible in high school. I shall say no more about this obvious fact. However, the mathematical needs of technical curricula must not be allowed to conceal the fact that this is just *one of many college curricula* where considerable use is found for secondary mathematics—and where some of this content is a prerequisite for *an efficient start in the freshman college year.* The following list of requirements in curricula *outside* the Institute of Technology at the University of Minnesota justifies the preceding remarks.

(1) *Premedical students.* The student is required to take two five-credit courses in mathematics, where advanced high-school algebra is a prerequisite. Premedical students in some types of advanced medical science are advised to take mathematics at least through elementary calculus.

(2) *Pharmacy curriculum.* The same ten-credit requirement as for the premedical curriculum.

(3) *Curricula in the School of Business.* Starting in the fall quarter of 1954, any student entering the School of Business, as a junior, will be required to present evidence that either (a) he took advanced algebra in high school, and then was able to pass a qualifying test in this content given at the University, or (b) he passed the substitute course, Mathematics 1, at the University. Also, ten credits in college mathematics, based on advanced high-school algebra as a prerequisite, are a requirement for majors in various curricula in the School of Business (for instance, Accounting and Insurance). Moreover, a desirable option in regard to a first course in Economic Statistics for sophomores will be made available, starting next year, to students who have credit in college algebra. The major curriculum in Statistics in Business requires the student to study college mathematics for three solid years.

(4) *College of Agriculture.* Certain curricula require college mathematics. In some curricula, substantial parts of college mathematics frequently are recommended, when not required.

(5) *College of Science, Literature, and the Arts.* Mathematical courses mentioned in items (1), (2), (3), and (4) are taught by the Department of Mathematics in this college. Major students of physics and chemistry are required to take at least two years of college mathematics with a premium placed on obtaining this knowledge as quickly as possible (aided by a strong entrance credit in mathematics). Students of proper abilities in the various statistical fields such as psychology, sociology, and economics are strongly advised to take a substantial amount of college mathematics. Finally I mention the large group of students majoring in mathematics. Approximately 70 senior college students at present are working toward a major in mathematics either in the College of Science, Literature, and the Arts, or under the wing of its Department of Mathematics as majors in mathematics in the College of Education.

(6) *Placement tests in mathematics in the College of Science, Literature, and the Arts.* In each course where advanced high-school algebra is a prerequisite, the student is required to pass a placement test in this content after a reasonable review. In each course where trigonometry is a prerequisite, credit in high-school trigonometry is acceptable if the student is able to pass a placement test in the subject. It should be noted that this test emphasized the analytical parts (identities, equations, graphs, etc.) as well as numerical trigonometry. Students who elect to take Math 1 in college as a substitute for advanced high-school algebra are given a placement test to justify continuation in

the course. Those students who fail on this test are given an opportunity to study very elementary algebra at a sub-freshman level in the Extension Division at an extra tuition charge.

In conclusion, I consider it obvious that a student who is able to learn advanced high-school mathematics to the point of mastery will be amply rewarded for his efforts when he reaches college.

Another view of the process of definition

by Robert S. Fouch, Tempe State College, Tempe, Arizona

In the April issue of this journal[1] Mr. Nichols presented a description of his method of teaching understanding of definition. It seems important to point out that in the history of western thought there have been many different views of this matter and that it has often been a subject of considerable philosophical controversy. I should like to describe here a view which I believe is more in keeping with contemporary conceptions of mathematics as well as with such recent philosophical developments as semantics and logical positivism.

We might start by asking: Which do we define, words or things? Mr. Nichols' use of the phrase "object being defined" suggests that he would subscribe to the view that we define things. The more mathematical view is, I believe, that definitions are about words and symbols and that they are, in fact, simply agreements to use a certain linguistic form in place of some other form which is usually larger or less convenient. To illustrate, suppose that we have discovered in geometry that certain triangles have two equal sides and that we have decided to study such triangles. After a while the frequent repetition of the phrase "triangle with two equal sides" becomes tiresome and we wonder if we might not save effort by using some shorter phrase. We then agree that we shall use the phrase "isosceles triangle" as an abbreviation for the phrase "triangle with two equal sides." This illustrates the basic process of definition and shows that it is a creative act, the act of inventing a new piece of language to take the place of some already existing words or symbols. This view makes clear the interchangeability of the *definiendum* (the new language) and the *definiens* (the old language). This has the further advantage of destroying all futile questions about "discovering" the meaning of a word and instead shows that there is only a question of historical fact, namely: for the replacement of what other language was this word, symbol, or phrase created?

In his fifth criterion, Mr. Nichols states that the converse of a good definition is true. This suggests that he may possibly ascribe truth or falsity to a definition. This also is denied in the view presented here, in which it would be said that when one makes a definition one is really saying, "I have decided to use this word in place of these other words." There can be no question about the fact of a decision having been made; there can only be a question about the wisdom or desirability of that decision. A good example is the definition of the phrase "natural number"; some writers include the number zero, others exclude it. There should be no claim that these definitions are contradictory nor any question which is correct. They are simply two different decisions about

[1] Eugene Nichols, "How I Teach Understanding of Definition," THE MATHEMATICS TEACHER, Vol. XLVII (April 1954), p. 274.

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NCTM

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Thirty-third Annual Convention

Hotel Statler, Boston, Massachusetts
April 13–16, 1955

HOST ORGANIZATION
New England Association of Mathematics Teachers

Marie S. Wilcox, Program Chairman
Thomas Carr Howe High School
Indianapolis, Indiana

One if by land!

Members of The National Council of Teachers of Mathematics will "invade" Boston from April 13 through April 16, 1955. Headquarters will be established at the Hotel Statler.

The time is long past when a teacher who knew the multiplication tables and remembered that there was a quadratic formula was considered capable of teaching a mathematics class. The past thirty years have seen considerable change in the topics taught in the mathematics class and in the manner in which they are presented. There may be a greater change in the next few years than in all those years put together!

Dr. G. Truman Hunter, Assistant to the Director of the Applied Science Division of the International Business Machines Corporation, in accepting the invitation to be the banquet speaker at the Boston meeting, stated, "Things are going very rapidly in the computing field, and by next spring we anticipate having considerably more information available on outstanding equipment and applications in the field of mathematics."

Dr. Harold Fawcett of Ohio State University, a well-known leader in the field of mathematics education, will address the opening general session on Thursday evening with a challenge to teachers to make pupils mathematically fit for life in this modern world.

Dr. E. P. Northrop of the University of Chicago will bring a message about modern mathematics to the group at Boston. Dr. Northrop is at present in Washington serving as Consultant in Education in the Sciences for the National Science Foundation. In this latter capacity he will participate in a panel to discuss recent efforts to strengthen education in the sciences. Other members of the panel will predict the influence on mathematics education of the 1954 summer conferences sponsored by the Ford Foundation, the National Science Foundation, and industry.

A message concerning the importance of mathematics will be brought to Council members at the Saturday morning general session by Dr. J. R. Killian, Jr., President of the Massachusetts Institute of Technology. Dr. Kenneth E. Brown, Specialist in Mathematics, United States Office of Education, will talk on professional opportunities in mathematics at a guidance section program on Friday afternoon.

Teachers of mathematics the country over should plan to attend the Boston

meeting to get a look into the future of mathematics and mathematics education as well as to exchange ideas on methods of teaching today.

Social events at the convention will include a reception following the Thursday evening program, the annual banquet on Friday, and the annual luncheon on Saturday.

A series of television broadcasts before and during the convention will feature the kinescopes described in the twenty-second yearbook of the Council. Dr. Phillip S. Jones of the University of Michigan, who produced these kinescopes, will describe the technique involved at a section meeting on television on Friday morning.

Short walking tours of historic Boston will highlight the sightseeing events.

Wednesday, April 13

Provision has been made for visits to schools in the Boston area on Wednesday. Sightseeing trips will be available.

Thursday, April 14

The Delegate Assembly will hold its first meeting Thursday morning. Meetings are also scheduled for State representatives, and for all Council committees. For those not concerned with these meetings, a sightseeing trip has been planned.

The afternoon will see a demonstration lesson and section programs at all levels of instruction. Programs concerned with research, supervision, adult education, and applications of mathematics will be of special interest.

Since schools in the Boston area are in session this week, a late afternoon program has been arranged so that teachers in the area may participate. At 3:45 P.M., laboratories in elementary, junior high, and senior high school mathematics are scheduled. There are also discussion groups and two formal programs at this hour.

The general meeting on Thursday evening will be held in the auditorium of the beautiful John Hancock building, two short blocks from the hotel.

Friday, April 15

Dr. Northrop will address the opening session on Friday morning. Papers and discussions concerning integrated mathematics, algebra, and geometry will be features of the day, with attention also given to guidance, evaluation, the use of television, the influence of summer conferences, and curriculum trends at the junior high school level.

The late afternoon program will be an attempt to acquaint all members of the Council with all Council activities. Officers, editors, and committee chairmen will make reports to the entire membership at 3:45 P.M., followed by the annual business meeting at 4:30 P.M.

Events of the day will be climaxed by the annual banquet and a message from Dr. Hunter.

Saturday, April 16

The general session will open with an address by Dr. Killian. Another feature of the morning program will be a panel discussion of the problems which confront the beginning teacher and is arranged to assist the prospective and beginning teacher. The Association of Teachers of Mathematics of New York will sponsor a program on Saturday afternoon concerning the problems of teaching mathematics as viewed by a classroom teacher, a department head, and a person in mathematics education.

Other programs during the day include discussions of desirable changes in the mathematics offered at the high school and college levels, teaching the gifted child, and the creation of a realization on the part of the child in junior high school that he should continue to study mathematics.

Section programs and discussion groups do not close until 4:00 P.M., so members can be governed by this in making hotel and transportation reservations.

• AFFILIATED GROUPS

H. Glenn Ayre, Chairman, Western Illinois State College, Macomb, Illinois

The Agenda Planning Committee has completed plans for the Sixth Delegate Assembly to be held as a part of the annual meeting of the National Council in Boston, April 13 to 16, 1955. The committee consists of Dr. Catherine A. V. Lyons, Pittsburgh, Pennsylvania, as chairman, assisted by a committee composed of Dr. Houston T. Karnes, Louisiana State University; Dr. Myron Rosskopf, Teachers College, Columbia University; Mr. Oscar F. Schaaf, Eugene, Oregon. In addition, valuable suggestions were made by Regional Representatives and others.

A slight change in procedure this year will make it possible to devote the entire first session to discussing problems posed by Affiliated Groups. A panel will present the problem of "Values of Affiliation" from the viewpoints of both the National Council and the Affiliated Groups. Discussion leaders will direct open discussions on the "Function and Selection of State Representatives" and "Small Group Affiliations."

It is suggested that each delegate be instructed by his group to be prepared with questions and suggestions on the following items:

1. What benefits should the Affiliated Groups expect from the National Council? What benefits should the Council expect from the Affiliates? What can be done to make these mutual benefits more effective?
2. What is the function of the State Representative? How selected and for how long?
3. Should small groups affiliate with the state organizations or directly with the National Council?
4. Recommendations to be considered by the Board. (Have typed copies for distribution.)

Mr. Henry Swain, Winnetka, Illinois, who has assumed the editorship of the *Handbook for Affiliated Groups*, expects to present the handbook in preliminary form. The Executive Committee of the Board agreed to ask that the first edition be in mimeographed form so that desired revisions could be made inexpensively before having it printed.

Reports from editors of THE MATHEMATICS TEACHER, *The Arithmetic Teacher*, *The Student Journal*, and reports by special committees, and other general reports will be made in conjunction with the annual business meeting. This will give delegates a better opportunity to gather information on the business of the Council to report back to their respective groups.

At the meeting of the First Delegate Assembly in Chicago, April 13 to 15, 1950, it was recommended that the Affiliated Groups have a definite part in planning and implementing the work of the National Council. This plan of mutual benefit initiated by the Assembly is proving to be a valuable force for the realization of the objectives of the Council. The Delegate Assembly is proving to be a valuable liaison between the Council and the local groups. Many recommendations proposed by the Assembly have been authorized and implemented by the Board.

It is hoped that all Affiliated Groups will make strong effort to be represented at this Sixth Delegate Assembly. If there is any group that has not selected a delegate and an alternate, this should be done

immediately and reported to the chairman of Affiliated Groups. This is our procedure for giving delegates official status.

In regard to renewal of affiliation and eligibility to vote in the Assembly the following action was taken by the Second Delegate Assembly and printed in the October 1951 issue of THE MATHEMATICS TEACHER.

Renew affiliation and pay dues each year between October 1 and December 31.

One vote be allowed each group regardless of size.

No delegate be allowed to vote at a Delegate Assembly if the annual dues for renewal of affiliation have not been paid.

Notes from the Washington office

For many years the registration reports of the Council conventions have been printed in THE MATHEMATICS TEACHER. These reports are the official record of the attendance at the meetings and thus become an important part of the history of the Council. Below are the reports for three recent meetings.

Registrations at the Fourteenth Summer Meeting

The National Council of Teachers of Mathematics, Seattle, Washington, August 22–25, 1954

Arizona	1	New Mexico	1
California	31	New York	3
Colorado	4	Ohio	3
District of Columbia	2	Oklahoma	4
Georgia	1	Oregon	25
Illinois	7	Pennsylvania	5
Indiana	2	South Dakota	1
Idaho	1	Tennessee	1
Kansas	3	Texas	2
Louisiana	1	Utah	3
Maryland	1	Virginia	3
Michigan	3	Washington	150
Minnesota	1	Wisconsin	4
Missouri	6	Wyoming	1
Montana	9	Foreign	4
Nebraska	4		
Nevada	2	Total	290
New Hampshire	1		

Registrations at the Annual Joint Meeting with the NEA

The National Council of Teachers of Mathematics, New York City, June 28, 1954

Alabama	3	Nebraska	1
Arkansas	4	New Jersey	36
California	1	New York	50
Colorado	1	North Carolina	2
Connecticut	1	Ohio	7
District of Columbia	6	Pennsylvania	7
Florida	1	Rhode Island	1
Georgia	1	South Carolina	2
Illinois	2	Tennessee	2
Indiana	2	Texas	2
Iowa	1	Virginia	10
Kansas	1	West Virginia	2
Louisiana	1	Wisconsin	1
Maryland	2	Foreign	1
Michigan	4		
Minnesota	2	Total	156

Continued on page 184

A column of unofficial comment

The National Council and classroom problems

by Marie Wilcox, President, The National Council of Teachers of Mathematics

At a time when our country needs more and more young people adequately trained in mathematics and science, we, unfortunately, also have a shortage of teachers capable of training these young people. Perhaps one way to improve this situation would be to give more attention to the problems of prospective and beginning teachers.

A few years ago I attended a meeting called to consider the problems of the classroom teacher. I was startled to hear experienced teachers tell of beginning teachers in their elementary schools who actually cried themselves to sleep at night because they just could not do everything each day which was necessary to carry on their work. They had children under their care the entire school day, often including their lunch period, and spent many hours after school on reports of many kinds, parent conferences, and preparation for special school events. Then far into the night they worked on plans just to carry on the classroom activities the next day. Beginning teachers in secondary schools often find themselves in a similar dilemma.

Many school systems are attempting to meet these problems by the use of consultants, in-service training conferences, and publications of various kinds. Teacher-training institutions hope to give their graduates as adequate preparation as possible to meet their problems as beginning teachers.

Professional organizations also have an obligation to these teachers!

The National Council of Teachers of Mathematics has attempted to keep its dues at a reasonable figure so that teachers in the lower salary brackets can afford to join. Student memberships at an even lower figure also are available for students who have never taught. Young men and women who are preparing to teach mathematics are not charged registration fees at the meetings of the Council if they are sponsored by a faculty member.

At the annual meeting in Boston from April 13 through 16 of this year, one Saturday morning section program will be directed to prospective and beginning teachers. A panel of young teachers will discuss problems they encountered in their first years of teaching. Henry Swain, a classroom teacher, will speak on the subject, "I Teach Mathematics and Like It." Discussion periods following the more formal portions of the program will afford the members of the audience an opportunity to present their own problems for discussion. The Hotel Statler, convention headquarters in Boston, offers a dormitory rate which will probably appeal to beginning teachers if a group of five or more are willing to room together.

In addition to the annual meeting of the Council, three other meetings are scheduled each year. The Place of Meeting Committee endeavors to recommend locations for these meetings so that a teacher in any part of the country may be able, without too great an expense for travel, to attend at least one meeting. As frequently as possible the summer and

Christmas meetings are housed in college or university dormitories. These meetings are during vacation periods so that it is possible for teachers to drive to the meetings. Rates for board and room when the meetings are housed in dormitories are very reasonable. The 1955 summer meeting which will be held at Indiana University offers board and room at $5 a day in double rooms and $5.50 in single rooms.

All publications of the Council contain much material which should be of assistance to the inexperienced teacher. This journal now gives further attention to the needs of these teachers in a department entitled, "Tips for Beginners," edited by Francis G. Lankford, Jr. A group of small publications is in preparation and will soon be available at reasonable prices. In this group is a "How to" series. Titles include: "How to Use Your Bulletin Board," "How to Study Mathematics," "How to Use Films and Film Strips," "How to Develop a Teaching Guide for Mathematics," and "How to Use Discovery Teaching Procedures in the Mathematics Classroom."

Actually hundreds of members of the Council, serving as officers, editors, and committee members, are making these meetings and publications possible. Prospective and beginning teachers can enjoy the benefits of this work only if they are informed of the assistance which the Council makes available to them.

Each member of the Council might well consider himself a committee of one to inform young men and women, who are prospective or beginning teachers of mathematics, of Council activities. It is of equal importance that he make suggestions to officers and committees of the Council concerning ways in which the organization might be of even greater assistance to these teachers.

Notes from the Washington office

Continued from page 182

Registrations at the Thirty-second Annual Meeting

The National Council of Teachers of Mathematics, Cincinnati, Ohio, April 20–24, 1954

Alabama	6	Missouri	7
Arizona	1	Nebraska	2
Arkansas	3	New Hampshire	1
California	4	New Jersey	16
Colorado	7	New York	36
Connecticut	5	North Carolina	4
Delaware	1	Ohio	296
District of Columbia	8	Oklahoma	2
Florida	9	Pennsylvania	24
Georgia	4	South Carolina	2
Illinois	55	Tennessee	12
Indiana	45	Texas	5
Iowa	8	Vermont	1
Kansas	2	Virginia	10
Kentucky	39	Washington	1
Louisiana	3	West Virginia	7
Maryland	8	Wisconsin	18
Massachusetts	11	Canada	6
Michigan	42	Foreign	1
Minnesota	9		
Mississippi	1	Total	722

Your professional dates

The information below gives the date, name, and place of meeting, with the name and address of the person to whom you may write for further information. For information about other meetings, see the previous issues of THE MATHEMATICS TEACHER. Announcements for this column should be sent at least ten weeks early to the Executive Secretary, The National Council of Teachers of Mathematics, 1201 Sixteenth Street N.W., Washington 6, D. C.

NCTM convention dates

April 13–16, 1955
ANNUAL MEETING
Statler Hotel, Boston, Massachusetts
Jackson Adkins, local chairman, Phillips Exeter Academy, Exeter, New Hampshire

July 4, 1955
JOINT MEETING WITH NEA
Chicago, Illinois
E. H. C. Hildebrandt, local chairman, Northwestern University, Evanston, Illinois

August 21–24, 1955
SUMMER MEETING
Indiana University, Bloomington, Indiana
Philip Peak, local chairman, Indiana University, Bloomington, Indiana

December 27–30, 1955
CHRISTMAS MEETING
Sheraton-Park Hotel, Washington, D. C.
Veryl Schult, local chairman, Wilson Teachers College, Washington 9, D. C.

Other professional dates

Second Annual Workshop of the Mathematics Teachers Association of Western Pennsylvania
March 19, 1955
University of Pittsburgh, Pennsylvania
Catherine A. V. Lyons, 12 S. Fremont Avenue, Pittsburgh 2, Pennsylvania

Eighth Annual Conference on the Teaching of Elementary and Secondary Mathematics
March 26, 1955
Illinois State Normal University, Normal, Illinois
T. E. Rine, ISNU Mathematics Conference, Illinois State Normal University, Normal, Illinois

Mathematics Conference of the Ontario Association of Teachers of Mathematics and Physics
April 13–14, 1955
University of Toronto, Toronto, Ontario
Ern Totton, Forest Hill Collegiate Institute, Toronto, Ontario

Fifth Annual Spring Conference of the Ohio Council of Teachers of Mathematics
April 29–30, 1955
Kent State University, Kent, Ohio
Mildred Keiffer, Cincinnati Public Schools, Cincinnati, Ohio

Annual Spring Conference of the Minnesota Council of Mathematics Teachers
April 29–30, 1955
Coffman Memorial, University of Minnesota, Minneapolis, Minnesota
Miss Angela Untereker, Technical High School, St. Cloud, Minnesota

Sixth Annual Conference of the Michigan Council of Teachers of Mathematics
April 29–May 1, 1955
St. Mary's Lake Camp, Battle Creek, Michigan
Irene Smith, Ann Arbor High School, Ann Arbor, Michigan

Spring Meeting of Colorado Council of Teachers of Mathematics
April 30, 1955
Pueblo Junior College, Pueblo, Colorado
Mrs. Amanda Lindsey, 753 S. Sherman Street, Denver 9, Colorado

Annual Meeting of Association of Mathematics Teachers of New York State
May 6–7, 1955
Hotel Syracuse, Syracuse, New York
Pauline A. Morris, Geneva High School, Geneva, New York

Annual Spring Meeting of the Indiana Council of Teachers of Mathematics
May 21, 1955
Butler University, Indianapolis, Indiana
R. L. Green, 3401 N. Meridian, Indianapolis, Indiana

University of Oklahoma Institute for Teachers of Mathematics
June 6–17, 1955
University of Oklahoma, Norman, Oklahoma
J. O. Hassler, University of Oklahoma, Norman, Oklahoma

National Science Foundation Collegiate Conference
June 13–July 22, 1955
Oklahoma A. and M. College, Stillwater, Oklahoma
L. Wayne Johnson, Oklahoma A. & M. College, Stillwater, Oklahoma

Sixth Annual Mathematics Institute
June 19–24, 1955
Louisiana State University, Baton Rouge, Louisiana
Houston T. Karnes, Louisiana State University, Baton Rouge 3, Louisiana

California Conference for Teachers of Mathematics
July 5–15, 1955
University of California, Los Angeles, California
Clifford Bell, University of California, Los Angeles 24, California

Third New Jersey Mathematics Institute
July 6–15, 1955
Rutgers University, New Brunswick, New Jersey
Director of the Summer Session, Rutgers University, New Brunswick, New Jersey

Third Annual Mathematics Institute of the Florida Council of Teachers of Mathematics
August 18–20, 1955
University of Florida, Gainesville, Florida
Kenneth P. Kidd, University of Florida, Gainesville, Florida

Seventh Annual Mathematics Institute of the Association of Teachers of Mathematics in New England
August 18–25, 1955
Middlebury College, Middlebury, Vermont
Harriet Howard, Ethel Walker School, Simsbury, Connecticut

Another view of the process of definition

Continued from page 178

the projected use of the phrase "natural number" and the reader is obligated to accept the writer's decision if he chooses to read on and to receive any communication.

Mr. Nichols seems to restrict his discussion to definition of nouns. I cannot say that this is actually restrictive because I suspect that with sufficient ingenuity one can twist almost any definition into the noun form; however, it does strike me that this may be highly artificial and very difficult in many cases. As a single example, consider the definition of the symbol "$\leqq$." It seems to me to be natural and simple to state: "$a \leqq b$" is defined as "$a = b$ or $a < b$." I am curious to see whether this can be defined within the framework of objects, classes, and characteristics.

On the matter of redundancy, I would agree that this is an undesirable characteristic but would add that this is occasionally difficult to detect in mathematics (and probably elsewhere) and that there is no danger of any serious logical consequences. As an example, suppose that someone has studied only a very little geometry and has become fascinated by the discovery that certain triangles have two equal sides and two equal angles. He may wish to make a definition at this point and I would not object to such a definition even though a theorem to be proved later will reveal the redundancy. The sixth criterion concerning previously known or defined terms would follow as a natural consequence of my position.

In conclusion, I should confess that when I first read Mr. Nichols' article, I was tempted to become quarrelsome and to try to arouse a controversy. Then I remembered that a mathematician should be consistent. Mr. Nichols has defined a good definition as something satisfying his six criteria; I have decided that the word "definition" will mean for me "a decision or agreement to use a new symbol interchangeably with some set of old symbols." We can quarrel only about the relative fruitfulness and convenience of our two definitions.

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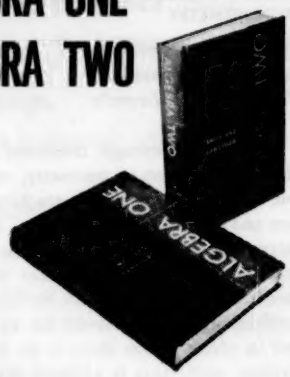

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